Topics for the Graduate Exam in Geometry and Topology

Most of the following topics are normally covered in the courses Math 535a and 540.

Differentiable manifolds: definition, submanifolds, smooth maps, tangent and cotangent bundles.

Differential forms: exterior algebra, integration, Stokes' theorem, de Rham cohomology.

Lie derivatives: of forms and vector fields.

Differential topology: regular values, Sard's theorem, degree of a map, and index of a vector field.

"Classical" differential geometry: local theory of surfaces, 1^{st} and 2^{nd} fundamental forms, Gauss-Bonnet formula.

Homotopy theory: definition of homotopy, homotopy equivalences, fundamental groups (change of base point, functoriality, Van Kampen theorem, examples such as the fundamental group of the circle), covering spaces (lifting properties, universal cover, regular (or Galois) covers, relation to π_1), higher homotopy groups.

Singular homology theory: definition of the homology groups, functoriality, relative homology, excision, Mayer-Vietoris sequences, reduced homology, connection between H_1 and the fundamental group, homology of classical spaces (e.g., S^n, $R^n - \{0\}$).

References:

M. Berger and B. Gostiaux: Differential Geometry: Manifolds Curves and Surfaces
A. Hatcher: Algebraic Topology
I.M. Singer and J.A. Thorpe: Lecture Notes on Elementary Topology and Geometry
H. Hopf: Differential Geometry in the Large, Springer Lecture Notes in Mathematics, V. 1000
M.J. Greenberg and J.R. Harper: Lectures on Algebraic Topology
J.W. Vick: Homology Theory
W.S. Massey: Algebraic Topology: An Introduction
I. Madsen and J. Tornehave: From Calculus to Cohomology