November 8, 2024 (Friday)
3:30pm-4:30pm
KAP 414

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Bounding adapted Wasserstein metrics

Abstract: The Wasserstein distance $\mathcal{W}_p$ is an important instance of an optimal transport cost. Its numerous mathematical properties as well as applications to various fields such as mathematical finance and statistics have been well studied in recent years. The adapted Wasserstein distance $\mathcal{A}\mathcal{W}_p$ extends this theory to laws of discrete time stochastic processes in their natural filtrations, making it particularly well suited for analyzing time-dependent stochastic optimization problems.

While the topological differences between $\mathcal{A}\mathcal{W}_p$ and $\mathcal{W}_p$ are well understood, their differences as metrics remain largely unexplored beyond the trivial bound $\mathcal{W}_p\lesssim \mathcal{A}\mathcal{W}_p$. This paper closes this gap by providing upper bounds of $\mathcal{A}\mathcal{W}_p$ in terms of $\mathcal{W}_p$ through investigation of the smooth adapted Wasserstein distance. Our upper bounds are explicit and are given by a sum of $\mathcal{W}_p$, Eder's modulus of continuity and a term characterizing the tail behavior of measures. As a consequence, upper bounds on $\mathcal{W}_p$ automatically hold for $\mathcal{AW}_p$ under mild regularity assumptions on the measures considered. A particular instance of our findings is the inequality $\mathcal{A}\mathcal{W}_1\le C\sqrt{\mathcal{W}_1}$ on the set of measures that have Lipschitz kernels.

Our work also reveals how smoothing of measures affects the adapted weak topology. In fact, we find that the topology induced by the smooth adapted Wasserstein distance exhibits a non-trivial interpolation property, which we characterize explicitly: it lies in between the adapted weak topology and the weak topology, and the inclusion is governed by the decay of the smoothing parameter.

This talk is based on joint work with Jose Blanchet, Martin Larsson and Jonghwa Park.