Simulation Methods for Stochastic Storage Problems: A Statistical Learning Perspective

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9th Western Conference on Mathematical Finance

Outline

Stochastic Storage: An optimal switching problem

- A storage problem has two key components:
  - Stochastic Risk factors (autonomous dynamics).
  - An inventory variable (fully controlled).
- Dynamics of the inventory is controlled via storage regimes (typically finite, > 2).
- In comparison, optimal stopping (American option pricing) has only two regimes: $\{\text{continue}, \text{stop}\}$ and stochastic factors.

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In today's talk we will discuss:

- A new modular algorithm (and its features) for optimal switching, combining Design of Experiments with Machine learning.
- Gaussian Process as a "basis-free" method for approximating continuation value.

Optimal switching problems

Gas storage valuation: A classic example

- Gas prices as stochastic factor with dynamics $P_{t_{k+1}} = P_{t_k} + b(t_k, P_{t_k})\Delta t + \sigma(t_k, P_{t_k})\,\Delta W_{t_k}$.
- Inventory variable: $I_{t_{k+1}} = I_{t_k} + a(c_{t_k})\,\Delta t$, $I_{t_k} \in [0, I_{\max}]$.
- Control c_{t_k} represents rate of injection/withdrawal or holding, fully determined by storage regime $m_{t_{k+1}} \in \{\text{inject}, \text{withdraw}, \text{hold}\}$.
- Objective: Create a policy $m_{t_{k+1}} = \mathcal{M}(t_k, P_{t_k}, I_{t_k}, m_{t_k})$ which maximizes the expected profit $v(t_0, \mathbf{P}_{t_0}, I_{t_0}, \mathbf{m}_{t_0})$ in the horizon $[0, T]$.

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Microgrid control

- Electric load and renewable output as stochastic factors.
- Battery storage system as inventory, controlled via backup diesel generator.
- Objective: Create a policy to match demand and supply of electricity and maximize the negative expected cost.

Value Function and DPE for Gas storage

- Value Function $V(t_k, P_{t_k}, I_{t_k}, m_{t_k}) =$

$$\sup_{\mathbf{m}_{t_k}} \mathbb{E}\left[\sum_{s=k}^{K-1} e^{-r(t_s - t_k)} \pi^{\Delta}(P_{t_s}, m_{t_s}, m_{t_{s+1}}) + e^{-r(T-t_k)} W(P_T, I_T) \Big| \, P_{t_k}, I_{t_k}, m_{t_k} \right],$$

where π^{Δ} incorporates the revenue and the switching cost.

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  where π^{Δ} incorporates the revenue and the switching cost.

- Dynamic Programming Equation

$$V(t_k, P_{t_k}, I_{t_k}, m_{t_k}) = \max_{m\in\mathcal{J}} \mathbb{E}\left[\pi^{\Delta}(P_{t_k}, m_{t_k}, m) + e^{-r\Delta t}V(t_{k+1}, P_{t_{k+1}}, I_{t_{k+1}}, m)\Big|P_{t_k}\right]$$

Value Function and DPE for Gas storage

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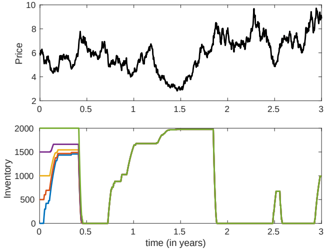
- Optimal Control

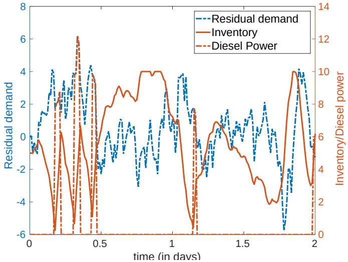
$$m^*(t_k, P, I, m) = \arg\max_{j\in\mathcal{J}} \left\{\pi^{\Delta}(P, m, j) + e^{-r\Delta t}q(t_k, P, I + a(c_{t_k}(j))\Delta t, j)\right\},$$

  where Continuation Value $q(t_k, P, I, m) := \mathbb{E}\left[V(t_{k+1}, P_{t_{k+1}}, I, m)\middle|\, P_{t_k} = P\right]$

A picture is worth a thousand words . . .



Gas Storage



Microgrid

A pathwise trajectory for two examples

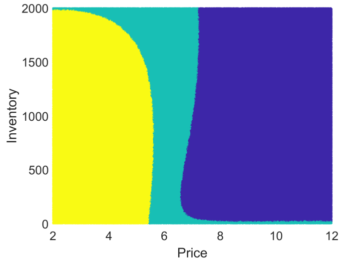
Gas Storage

- Buy low and sell high.

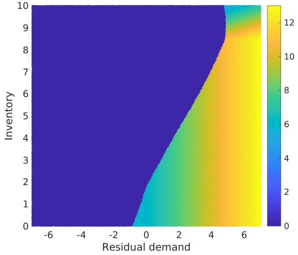
Microgrid

- Switch ON/OFF the generator to match demand and charge battery.

Control Maps



Gas Storage $m^*(P, I, +1)$



Microgrid $c^*(X, I, +1)$

Optimal Policy

How to efficiently estimate the maps $m^*(P, I, m)$?

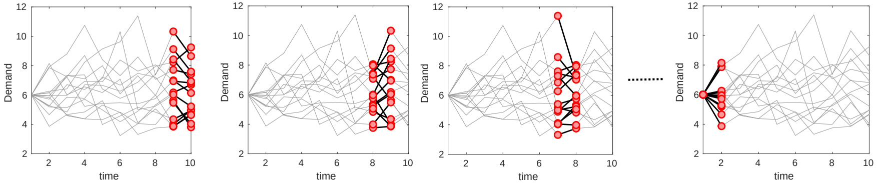
Contributions

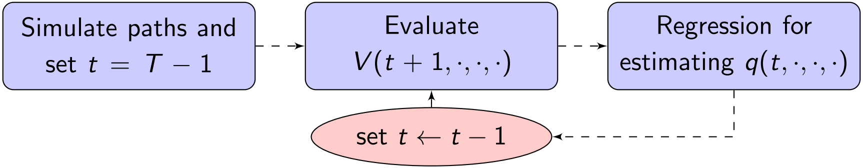
- A new algorithm Dynamic Emulation Algorithm(DEA)
  - Flexible modular template by reformulating optimal switching problem as recursive statistical learning.
  - Memory efficient and Scalable via "smart" simulation/regressions.
- Several new simulation designs which are dynamic, explores the space and exploits the distributional content.
- Gaussian Process Regression: a non-parametric tool for approximating the continuation function $q(t_k, \cdot, \cdot, m)$

Outline

1. Stochastic storage
2. Dynamic Emulation Algorithm
3. Numerical Illustrations
4. Conclusion

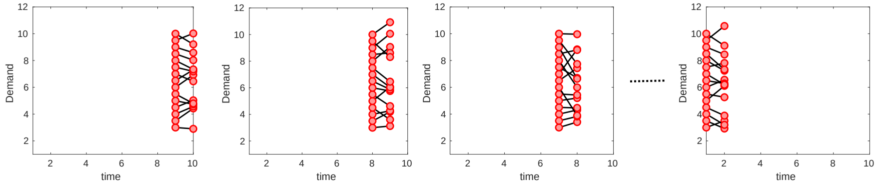
Traditional Implementation

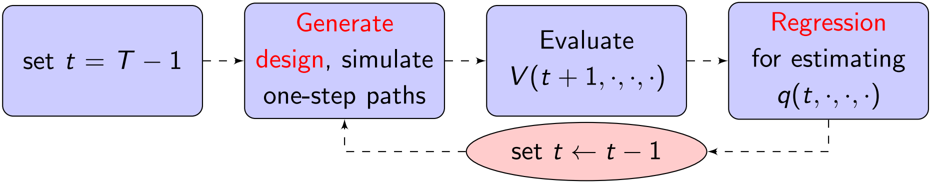




- Paths for the exogenous process is simulated and stored for [0,T].
- Controlled trajectory I_t cannot be simulated. So, replicate the exogenous process P_t/X_t for all possible values of I_t.
- Non-parametric regression methods are very slow since we need $\approx 10^4$ paths for reasonable estimation.

Dynamic Emulation Algorithm (DEA)

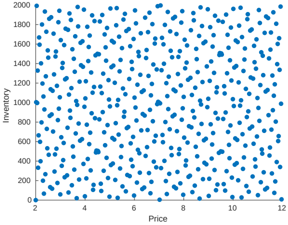




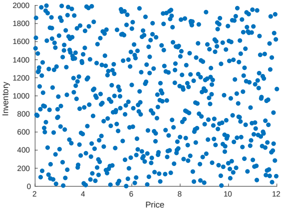
- Stochastic simulation for sequential learning of q.
- Both design and regression affects the quality of the solution.
- "Smart" design ☛ Improved learning.

Design

- Space Filling Design
  - Sobol QMC Sequences: Theoretical guarantees on “uniform” space filling.
  - Latin Hypercube Sampling (LHS): Probabilistic counterpart of QMC sequences.



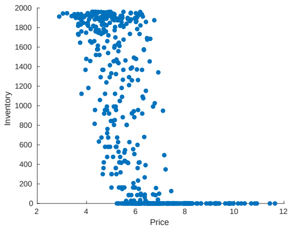
2D Sobol QMC sequence



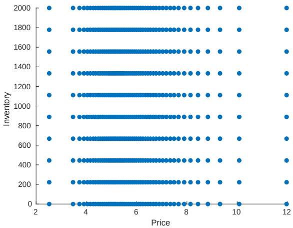
2D LHS design

Design

- Probabilistic Design: Mimics the distribution of (P, I). Too targeted, fails to explore the space.
- Gridded Design: Probabilistic in P and equally spaced in I



Joint Probabilistic design



Conventional product design

Design

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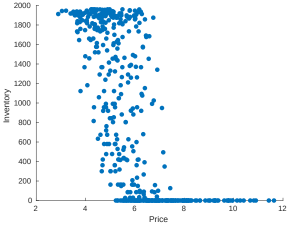

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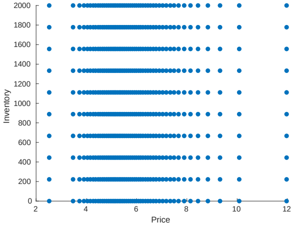

Conventional product design

- Mixture Design: Combination of space filling and probabilistic designs (exploration-exploitation tradeoff).

Stochastic Simulator (at time t_k)

- Choose the design $\mathcal{D}_k := (P^n_{t_k}, I^n_{t_{k+1}}, m^n_{t_{k+1}}, n = 1, \ldots, N_k)$.
- Generate one-step paths $P^n_{t_k} \mapsto P^n_{t_{k+1}}$.
- Calculate one-step-ahead pathwise profits:

$$v^n_{k+1} := \max_{j \in \mathcal{J}} \left\{ \pi^{\Delta}(P^n_{t_{k+1}}, m^n_{t_{k+1}}, j) + e^{-r\Delta t} \hat{q}(t_{k+1}, P^n_{t_{k+1}}, I^n_{t_{k+1}} + a(c(j))\Delta t, j) \right\}$$

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- Learn the input-output relationship between $(P_{t_k}, I_{t_{k+1}}, m_{t_{k+1}})_{n=1}^{N_k}$ and $(v_{k+1})_{n=1}^{N_k}$ using L^2 projection on an approximation space $\mathcal{H}_k$.

$$\hat{q}(t_k, \cdot, \cdot, \cdot) = \arg\min_{h_{t_k} \in \mathcal{H}_k} \sum_{n=1}^{N} \left| h_{t_k}(P^n_{t_k}, I^n_{t_{k+1}}, m^n_{t_{k+1}}) - v^n_{k+1} \right|^2.$$

Example: $\mathcal{H}_k = \mathrm{span}(\phi_1, \dots, \phi_R)$ and $h_{t_k}(\cdot) = \sum_{i=1}^{R} \beta_i \phi_i(\cdot)$

- We use Gaussian Process as a new alternate for $\mathcal{H}$.

Gaussian Process Regression (GPR)

- Assuming that the input-output relationship as

$$y^n = h(x^n) + \sigma^2 \xi,$$

where $\xi \sim \mathcal{N}(0,1)$, $x^n = (P^n_{t_k}, I^n_{t_{k+1}})$ and $y^n = v^n_{k+1}$.
- We assume $h(\cdot)$ is a realization of Gaussian random field.
- $\{h(x_i)\}_{i=1}^N$ is a sample from the multivariate normal distribution (MVND) with mean $\{m(x_i)\}_{i=1}^N$ and covariance $\{\kappa(x_i, x_j)\}_{i,j=1}^N$.
- Conditional distribution at new sites $\mathbf{x}_*$ is also multivariate normal i.e. $h(\mathbf{x}_*)|(x_i, y_i)_{i=1}^N \sim MVND(m_*(\mathbf{x}_*), \kappa_*(\mathbf{x}_*, \mathbf{x}_*))$

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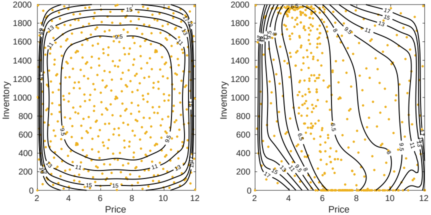
Advantages of using GPR:

- Analytic tractability for a non-parametric method.
- Smooth approximation for the function and even its derivative.
- Standard error on estimates at any new location x.

Gaussian Process Regression

- Rectangular level sets for the standard error due to space filling design (Sobol QMC).
- Mixture design leads to lower standard error in the switching region.



Impact of design on posterior standard error. Left: Sobol QMC, Right: Mixture design

Allows for sequential design (Gramacy and Ludkovski (2015), Ludkovski (2018) and Hu and Ludkovski (2016)).

Outline

Example1: Gas Storage

Design	Regression Scheme	Simulation Budget Low	Medium	Large
Conventional	PR-1D	4,965	5,097	5,231
	GP-1D	4,968	5,107	5,247
Adaptive 1D	PR-1D	5,061	5,187	5,246
	GP-1D	5,079	5,195	5,245
Dynamic	GP-1D	5,132	5,225	5,266
	Mixed	5,137	5,205	5,228
Mixture 2D	PR-2D	4,820	4,835	4,834
	GP-2D	5,137	5,210	5,233

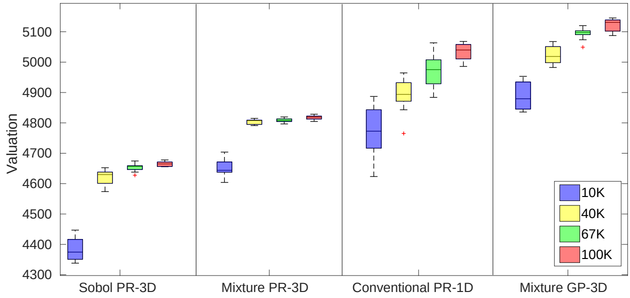
Valuation $\hat{V}(0, 6, 1000)$ (in thousands) using different design-regression pairs and three simulation budgets: Low $N \simeq 10K$, Medium $N \simeq 40K$, Large $N \simeq 100K$. PR: Polynomial regression, GP: Gaussian process

- Dynamic design: Varying budget, particularly higher simulation budget at the boundary due to penalty.
- Dynamic design + Mixed Regression: Varying budget and regression scheme as we move through the backward induction.
- Mixture design: 60% probabilistic and 40% space filling design.

☛ Higher valuation with mixture and dynamic design compared to conventional methods.

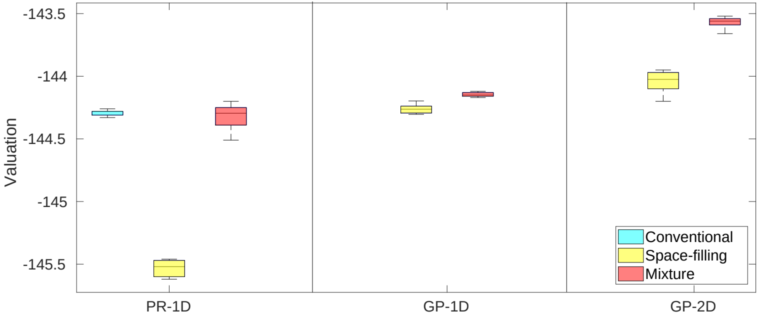
Example2: Double Gas Storage

Jointly estimate the valuation of two independent gas storage facilities. State space (P_t, I_t^1, I_t^2), where I_t^j is the inventory for j^{th} facility.



☛ Space Filling design < Conventional design < Mixture design.
☛ Gaussian process better than the state-of-the-art 1D regressions.
☛ Parametric 1D regressions significantly outperform parametric -3D irrespective of the design.

Example3: Microgrid



☛ Poor performance of space filling designs across regression methods, GP more robust than the state-of-art.

☛ GP-2D with mixture design substantially better than Parametric -1D.

Conclusion

So far . . .

- We developed a new Dynamic Emulation Algorithm:
  - Scalable (very low memory requirement).
  - Flexible (vary budget/regression/design distribution across time-steps).
  - Fully controlled processes can be handled with ease (work in progress)
- Non-parametric regression methods for learning the value function.
- Emphasis on "smart" design through several examples exhibiting dynamic, exploratory and exploitative aspects of the algorithm.
- See `https://arxiv.org/abs/1803.11309` for Microgrid example and other details.

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Work in progress

- We extend Dynamic Emulation Algorithm for stochastic optimal control with probabilistic constraints.

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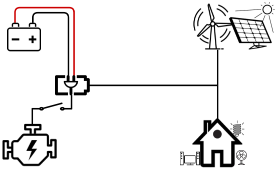
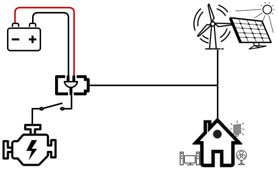
Microgrid Management



- Net demand (Load - Renewables) as stochastic factor with dynamics $X_{t_{k+1}} - X_{t_k} = \alpha(\underline{X} - X_{t_k})\Delta t + \sigma \Delta W_{t_k}$.
- Inventory variable: $I_{t_{k+1}} = I_{t_k} + a(c_{t_k})\Delta t = I_{t_k} + B_{t_k}\Delta t,\ I_{t_k} \in [0, I_{\max}]$.
- Control of diesel generator $c_{t_k}(1) = X_{t_k}\mathbf{1}_{\{X_{t_k}>0\}} + B_{\max} \wedge \frac{I_{\max} - I_{t_k}}{\Delta t}$.
- Regime $m_{t_{k+1}} \in \{\text{Generator ON}, \text{Generator OFF}\}$.

Microgrid Management



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- Regime $m_{t_{k+1}} \in \{\text{Generator ON}, \text{Generator OFF}\}$.
- Battery $B_{t_k} := a(c_{t_k}) = -\frac{I_{t_k}}{\Delta t} \vee \left(B_{\min} \vee (c_{t_k} - X_{t_k}) \wedge B_{\max}\right) \wedge \frac{I_{\max} - I_{t_k}}{\Delta t}$.
- One step profit $\pi(c, X) := -c^{\gamma} - |S|\Big[C_2\mathbf{1}_{\{S<0\}} + C_1\mathbf{1}_{\{S>0\}}\Big]$, where $S_{t_k} = c_{t_k} - X_{t_k} - B_{t_k}$.
  - $S_{t_k} < 0$ leads to blackout; $S_{t_k} > 0$ waste of energy.

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Design	Regression Scheme	Simulation Budget Low	Medium	Large
Conventional	PR-1D	4,965	5,097	5,231
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	PR-2D	4,869	4,888	4,891
	LOESS-2D	4,910	4,969	5,011
	GP-2D	4,652	5,161	5,243
Space-filling	PR-1D	4,768	4,889	5,028
	GP-1D	4,854	5,064	5,224
	PR-2D	4,762	4,789	4,792
	LOESS-2D	4,747	4,912	4,934
	GP-2D	4,976	5,080	5,133
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Valuation $\hat{V}(0, 6, 1000)$ (in thousands) using different design-regression pairs and three simulation budgets: Low $N \simeq 10K$, Medium $N \simeq 40K$, Large $N \simeq 100K$. The valuations are averages across 10 runs of each scheme except for LOESS-2D with large budget: due to the excessive overhead of LOESS only a single run was carried out.