The coordination of centralised and distributed electricity generation

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Introduction

1. Three optimization problems
   1.1. The consumer
   1.2. The energy company
   1.3. The social planner

2. Looking for an equilibrium

Conclusions
We focus on a major change occurred in energy markets over the last few years: the coordination of centralised and distributed generation.

Development of affordable distributed energy sources (solar panels)

⇓

Consumers can now produce by themselves a certain amount of electricity and then buy in the market the energy they still need

⇓

New interactions between energy companies and final consumers (no longer unidirectional system, now bidirectional system)

⇓

Need for new models
We here consider the point of view of:

- a representative consumer, who self-produces energy by solar panels and faces relevant installation costs. *How many panels to install?*
- a representative energy company, who needs to adapt its production strategy to the consumer’s decisions. *How much energy to produce?*
- a social planner, who wants to minimize the global costs. *Which strategies would he suggest to the consumer/company?*
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- a representative consumer, who self-produces energy by solar panels and faces relevant installation costs. How many panels to install?
- a representative energy company, who needs to adapt its production strategy to the consumer’s decisions. How much energy to produce?
- a social planner, who wants to minimize the global costs. Which strategies would he suggest to the consumer/company?

Our goals: finance. Solution to the three problems above? Equilibrium between the planner’s suggestions and the consumer/company’s choices?

Our goals: mathematics. Solution to linear-quadratic McKean-Vlasov control problems with stochastic coefficients?
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- Let $\alpha_t$ be the number of panels the consumer buys/sells in $t$ and let $dX_t^\alpha = b\alpha_t dt + \sigma X_t^\alpha dW_t$ be the energy the panels produce in $t$.

- Buying/selling panels has a cost, quadratic w.r.t. $\alpha_t$: $c\alpha_t + \gamma \alpha_t^2$. 

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- Buying/selling panels has a cost, quadratic w.r.t. $\alpha$: $c\alpha_t + \gamma \alpha_t^2$.

- If $D$ is the consumer’s electricity demand (constant), $D - X_t^{\alpha}$ is the amount of electricity still needed and bought in the market, at price $P_t + \theta$ ($\mathcal{F}^{W^0}$-adapted, $W^0 \perp W$, $\theta$ transp. cost). No model on $P$. 


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- As the consumer wants a stable production of energy from solar panels, the variance of the production $\text{Var}[X_t^\alpha]$ is penalized.
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\mathbb{E} \left[ \int_{0}^{\infty} e^{-\rho t} \left( c \alpha_t + \gamma \alpha_t^2 + (P_t + \theta)(D - X_t^\alpha) + \eta \text{Var}[X_t^\alpha] \right) dt \right]
$$
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So, the consumer has to solve

$$\inf_{\alpha} E \left[ \int_0^\infty e^{-\rho t} \left( c\alpha t + \gamma \alpha^2 t + (P_t + \theta)(D - X_t^\alpha) + \eta \text{Var}[X_t^\alpha] \right) dt \right],$$

$$dX_t^\alpha = b\alpha_t dt + \sigma X_t^\alpha dW_t, \quad P \text{ stochastic.}$$

Notice: linear quadratic McKean-Vlasov control problem.
Consumer: optimal control. After precise computations, the optimal control $\hat{\alpha}$ is $(K, \Lambda > 0$ explicit, $\hat{X} := X^{\hat{\alpha}}$)

$$
\hat{\alpha}_t = - \frac{bK}{\gamma} (\hat{X}_t - \mathbb{E}[\hat{X}_t])
+ \frac{b}{2\gamma} \int_t^\infty e^{-(\rho+b^2K/\gamma)(s-t)} \mathbb{E}[P_s | \mathcal{F}_t^0] ds
+ \frac{b}{2\gamma} \int_t^\infty \left( e^{-(\rho+b^2\Lambda/\gamma)(s-t)} - e^{-(\rho+b^2K/\gamma)(s-t)} \right) \mathbb{E}[P_s] ds
- \frac{b\Lambda}{\gamma} \mathbb{E}[\hat{X}_t] + \frac{b\theta - \rho c}{2\gamma(\rho + b^2\Lambda/\gamma)}.
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- \frac{b\Lambda}{\gamma} \mathbb{E}[\hat{X}_t] + \frac{b\theta - \rho c}{2\gamma (\rho + b^2\Lambda/\gamma)}.
\]

If there exists $\bar{P} := \lim_t \mathbb{E}[P_t]$, then the average number of panels and production get constant:

\[
\lim_{t \to \infty} \mathbb{E}[\hat{\alpha}_t] = 0, \quad \lim_{t \to \infty} \mathbb{E}[\hat{X}_t] = \frac{\bar{P} + \theta - \rho c}{2\sigma^2K} =: \hat{X}_\infty.
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**Consumer: simulations.** We now see the effect of penalizing the variance $(dP_t = \vartheta(\bar{P} - P_t)dt + \xi dW_t, \text{OU proc.})$: 
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Finally, the convergence of the average production $\mathbb{E}[\hat{X}_t]$ as $t \to \infty$. 
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![Trajectories of X](image-url)
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- Let \( \nu_t \) be the production installation rate in \( t \) (\( \nu_t > 0 \) improves the production) and let \( dQ^\nu_t = \nu_t \, dt \) be the energy produced in \( t \).

- Modifying the prod. rate has a cost, quadratic w.r.t. \( \nu_t \): \( h\nu_t + \delta\nu_t^2 \).
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- As the quantity $Q^\nu_t$ should correspond to $D - X_t^\alpha$, there is a penalty in case of overproduction or underproduction.
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$$dQ_t^\nu = \nu_t dt, \quad P_t, X_t^\alpha \text{ stochastic.}$$
Company: optimal control. We solve the problem as above. Let \( \hat{\nu} \) be the optimal control; we have (\( \tilde{K} > 0 \) explicit, \( \hat{Q} = Q^{\hat{\nu}} \)):

\[
\hat{\nu}_t = -\frac{\tilde{K}}{\delta} \hat{Q}_t (X^\alpha) + \frac{\lambda}{\delta} \int_t^\infty e^{-(\rho+\tilde{K}/\delta)(s-t)} \mathbb{E}[D - X_s^\alpha | \mathcal{F}_t] ds - \frac{\pi + h\rho}{2\delta(\rho + \frac{\tilde{K}}{\delta})}.
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Second term: (over)discounted energy expected to be sold in \([t, \infty[\).
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Social planner: problem. He minimizes the sum of the two payoffs:

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\inf_{(\alpha, \nu)} \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \left( c\alpha_t + \gamma \alpha_t^2 + h\nu_t + \delta \nu_t^2 + \theta (D - X_t^\alpha) + \pi Q_t^\nu + \eta \text{Var}[X_t^\alpha] + \lambda (D - X_t^\alpha - Q_t^\nu)^2 \right) dt \right],
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$$dX_t^\alpha = b\alpha_t dt + \sigma X_t^\alpha dW_t, \quad dQ_t^\nu = \nu_t dt,$$

$P$ stochastic.

Social planner: optimal control. Explicit expressions for the opt. cont. $(X^*, Q^*)$ are available (...but complicated!). The limits are:

$$\lim_{t \to \infty} \mathbb{E}[\alpha_t^*] = 0, \quad \lim_{t \to \infty} \mathbb{E}[X_t^*] = \frac{\rho h + \pi + \theta - \frac{\rho c}{b}}{2\sigma^2 K^{11}} =: X_{\infty}^*,$$

$$\lim_{t \to \infty} \mathbb{E}[\nu_t^*] = 0, \quad \lim_{t \to \infty} \mathbb{E}[Q_t^*] = D - X_{\infty}^* - \frac{\pi + \rho h}{2\lambda} =: Q_{\infty}^*.$$
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Pareto: a suitable definition. Recall the results of the three pbs.

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Definition (first attempt). A Pareto equilibrium is a price process $P$ such that $\hat{X}_t(P) = X^*_t$ and $\hat{Q}_t(\hat{X}(P)) = Q^*_t$, for $t \geq 0$. 
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**Definition (first attempt).** A Pareto equilibrium is a price process $P$ such that $\hat{X}_t(P) = X^*_t$ and $\hat{Q}_t(X^*) = Q^*_t$, for $t \geq 0$.

Second one: true (proof as above). We focus on the first one: very hard to solve. Idea: weaker definition, in term of limits.

**Definition (second attempt).** An asymptotic Pareto equilibrium is a real number $\bar{P}$ such that $\lim_{t \to \infty} \mathbb{E}[\hat{X}_t](\bar{P}) = \lim_{t \to \infty} \mathbb{E}[X^*_t]$. 
Indeed, some admissibility conditions are necessary...

**Definition.** An admissible asymp. Pareto equilibrium is a real $\bar{P}$ s.t.
\begin{itemize}
  \item $\lim_t \mathbb{E}[\hat{X}_t](\bar{P}) = \lim_t \mathbb{E}[X^*_t]$;
  \item $\lim_t \mathbb{E}[X^*_t] \in ]0, D[,$ $\lim_t \mathbb{E}[Q^*_t] \in ]0, +\infty[,$ $\bar{P} \in ]0, +\infty[.$
\end{itemize}
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**Definition.** An admissible asymp. Pareto equilibrium is a real $\bar{P}$ s.t.

- $\hat{X}_\infty(\bar{P}) = X^*_\infty$;
- $X^*_\infty \in ]0, D[,$ $Q^*_\infty \in ]0, +\infty[,\quad \bar{P} \in ]0, +\infty[.$
Indeed, some admissibility conditions are necessary...

**Definition.** An admissible asymptotic Pareto equilibrium is a real $\bar{P}$ s.t.
- $\hat{X}_\infty(\bar{P}) = \bar{X}^\ast$;
- $\bar{X}^\ast_\infty \in ]0, D[$, $\bar{Q}^\ast_\infty \in ]0, +\infty [$, $\bar{P} \in ]0, +\infty [$.

**Pareto: conditions and formulas.** We now look for admissible asymptotic Pareto equilibria for our problem. The equation $\hat{X}_\infty(\bar{P}) = \bar{X}^\ast_\infty$ corresponds, by the formulas above, to

$$\bar{P} + \theta - \frac{\rho c}{b} = \frac{\rho h + \pi + \theta - \frac{\rho c}{b}}{2\sigma^2 K^{11}}.$$ 

We solve w.r.t. $\bar{P}$ and check the three admissibility conditions.
Proposition

We can provide necessary and sufficient conditions for the existence of an admissible asymptotic Pareto equilibrium. In this case, the equilibrium is unique and defined as:

\[ \bar{P}^{Par} = \left(1 - \frac{K}{K^{11}}\right)\left(\frac{\rho c}{b} - \theta\right) + \frac{K}{K^{11}}(\rho h + \pi). \]
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We can provide necessary and sufficient conditions for the existence of an admissible asymptotic Pareto equilibrium. In this case, the equilibrium is unique and defined as:

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Some remarks about the previous result.

- The $\exists!$ condition is very mild: just $\rho c/b - \theta \leq \rho h + \pi$.
- We have $K, K^{11} > 0$ and it can be proved that $K < K^{11}$, so that $0 < \frac{K}{K^{11}} < 1$. Then, the equilibrium $\bar{P}_{\text{Par}}$ above can be seen as a convex linear combination.
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1. Optimal production strategies

- Consumer’s demand satisfied by self-production and market
- Point of view of a consumer, a company, a social planner
- Framework: McKean-Vlasov stochastic optimal control
- Explicit formula for the optimal controls

2. Looking for an equilibrium

- Definition of admissible asymptotic Pareto equilibrium
- Necessary and sufficient conditions for existence and uniqueness
- Explicit formula for the equilibrium
...some self-advertisement to conclude!

Joint work with Haoyang Cao and Xin Guo (UC Berlekey), soon in ArXiv.

- \( N \)-player stochastic games with impulse controls.
- Mean-field stochastic games with impulse controls.
- Application to the Constantinides-Richard cash management problem: 1 vs. 2 vs. \( \infty \) players.
Thank you!