Optimal Dividend Distribution Under Drawdown and Ratcheting Constraints on Dividend Rates

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9^{th} Western Conference on Mathematical Finance

University of Southern California, November 2018

UNIVERSITY of WASHINGTON

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  - Case I: dividend rate is restricted to $[0, C_0]$
    Pay dividends at rate 0 if surplus is lower than some value X^* and at rate C_0 if surplus is greater than X^*
  - Case II: dividend rate is unrestricted
    Payout any surplus in excess of a barrier b

- Following a barrier strategy results in a volatile all-or-nothing path for dividend stream
- Shareholders and analysts react negatively (and arguably overreact) when the rate of dividend payment decreases
- We propose a way to smooth the rate of dividend payments by requiring that the rate of dividend payments to never fall below a fraction of its historical maximum rate
- This requirement is different from most drawdown constraints in the literature, which apply to the surplus or wealth itself
  e.g. Grossman and Zhou (1993), Cvitanić and Karatzas (1995), and Elie and Touzi (2008)

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- A company is deciding on its investment and dividend policies

- Investment policy:
  - For simplicity, we assume the number of shares to be fixed only bonds are issued or bought back
  - π_t: total asset at t
  - X_t: total equity at t, also called the "surplus"
  - $\pi_t - X_t$: total debt

- Dividend policy: dividend is paid at rate C_t \$/yr from the surplus
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- Dividend policy is subject to two constraints

  (i) The dividend rate must be higher than the interest rate

  $$c_t := C_t - rX_t > 0; \quad t \geq 0$$

  (ii) The drawdown constraint: $c_t \geq \alpha\, z_t; \quad t \geq 0$

  $z_t := \max\left\{z, \sup_{0 \leq s < t} c_s\right\}$, $z > 0$ and $0 < \alpha \leq 1$

  Dividend rate is allowed to increase beyond its historical peak in which case $c_t > z_t$

- Surplus evolves according to

$$\mathrm{d}X_t = \pi_t \frac{\mathrm{d}I_t}{I_t} + \Big(r(X_t - \pi_t) - C_t\Big)\mathrm{d}t = \Big(\mu\pi_t - c_t\Big)\mathrm{d}t + \sigma\pi_t\,\mathrm{d}W_t,$$

where $\{I_t\}$, the "intrinsic value" of the company, is a GBM

$$\frac{\mathrm{d}I_t}{I_t} = (\mu + r)dt + \sigma \mathrm{d}W_t$$

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Problem Setup

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$(W_t)_{t\geq 0}$ is a Brownian motion on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$

- $(\pi_t, c_t)_{t\geq 0}$ is *admissible* if:
  - (i) $(\pi_t)_{t\geq 0}$ is $(\mathcal{F}_t)$-progressively measurable and satisfies $\pi_t \geq 0$ and $\int_0^\infty \pi_t^2 \mathrm{d}t < \infty$, $\mathbb{P}$–almost surely
  - (ii) $(c_t)_{t\geq 0}$ is $(\mathcal{F}_t)$-adapted, non-negative, and right-continuous with left limits; and
  - (iii) $c_t \geq \alpha\, z_t, t \geq 0$, where $z_t := \max\Big\{z, \sup_{0\leq s<t} c_s\Big\}$

- $\mathbb{C}(\alpha, z)$: the set all admissible policies

- Objective:

$$\sup_{(\pi_t,c_t)\in\mathbb{C}(\alpha,z)} \mathbb{E}\left[\int_0^{\tau} \mathrm{e}^{-\delta t}\,\frac{c_t^{1-p}}{1-p}\,\mathrm{d}t\right].$$

- $\tau = \tau^{X^{(\pi_t,c_t)}} := \inf\left\{t \geq 0 :\ X_t \leq 0\right\}$ is the time of bankruptcy
  For $\alpha, z > 0$, we have $c_t > 0$ and thus bankruptcy is "not avoidable"
  $\mathbb{P}(\tau < \infty) > 0$

- $\delta > 0$: subjective time preference, larger values indicates more impatient shareholders

- p is the constant relative risk aversion, satisfying $\frac{1}{\frac{2\,\sigma^2}{\mu^2}\,\delta+1} < p < 1$, reason:
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  - $p \geq 1$: How to "penalize" bankruptcy? Two suggestions:
    $\int_0^\tau e^{-\delta t}\,\frac{c_t^{1-p}}{1-p}\,dt \longrightarrow \tau < \infty$ is rewarded! Immediate liquidation is optimal
    $\int_0^\infty e^{-\delta t}\,\frac{c_t^{1-p}}{1-p}\,dt \longrightarrow \tau < \infty$ is penalized by $-\infty$, the problem is infeasible
- Note that $\frac{c_t^{1-p}}{1-p} \to \infty$ as $p \to 1$

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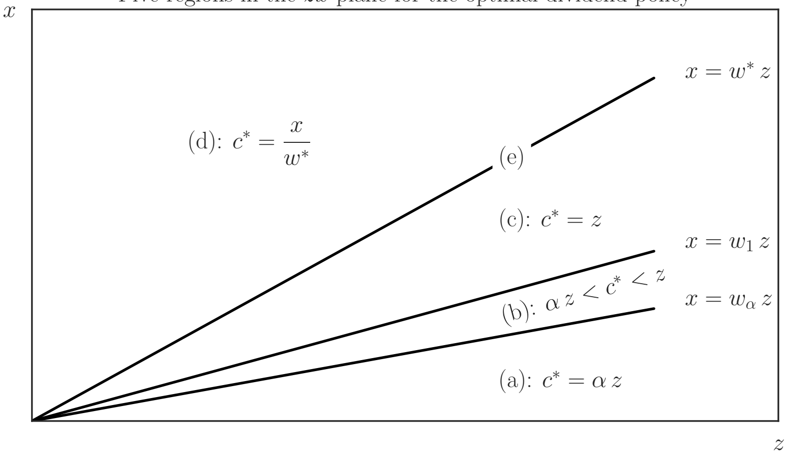
- The optimal dividend policy depends on (X_t) and (z_t) or, more specifically, the value of the "surplus-to-historical peak" ratio X_t/z_t
- There exist constants $0 < w_\alpha < w_1 < w^*$ such that:
  (a) If $X_t < w_\alpha z_t$, then $c_t = \alpha z_t$
  (b) If $w_\alpha z_t < X_t < w_1 z_t$, then $c_t = c^*(X_t, z_t) \in (\alpha z_t, z_t)$, for some function $c^*(x, z)$
  (c) If $w_1 z_t \leq X_t < w^* z_t$, then $c_t = z_t$
  (d) If $X_t > w^* z_t$, then $c_t = \frac{X_t}{w^*} > z_t$
  In this case, the historical peak has a jump at t, that is, $\lim_{s \to t^+} z_s = \frac{X_t}{w^*} > z_t$
  (e) Along the line $x = w^* z$, the company increases its dividend rate via singular control to keep $X_t \leq w^* z_t$

Five regions in the zx-plane for the optimal dividend policy



- (z_t) can have a jump only at time $t = 0$ and only if rule (d) is applicable, that is, $X_0 > w^* z_0$
- Afterwards, the process $(X_t, z_t)_{t \geq 0}$ will be kept in the domain
$$\mathcal{D} = \{(x, z) : 0 \leq x \leq w^* z, z > 0\}$$
- In particular, (z_t) is only allowed to increase via singular control in order to keep (X_t, z_t) inside $\mathcal{D}$

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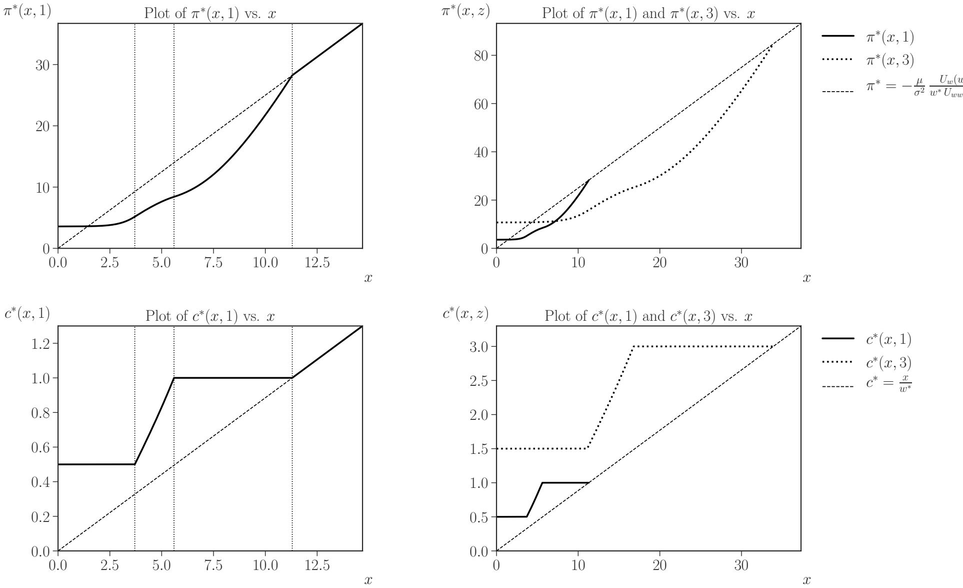
- In particular, (z_t) is only allowed to increase via singular control in order to keep (X_t, z_t) inside $\mathcal{D}$
- As a consequence of the optimal policy, $M_t^* = w^* z_t; \quad t > 0$

  where $M_t^* := \max \left\{ M_0, \ \max_{0 \leq s < t} X_s^* \right\}$ and $M_0^* = w^* z_0$

  The running max of (the optimally controlled) surplus is proportional to the historical consumption peak

Plots of the Optimal Policy



- Value function: $V(x,z) = \sup_{(\pi_t, c_t)\in \mathbb{C}(\alpha,z)} \mathbb{E}^{\,x}\left[\int_0^\tau \mathrm{e}^{-\delta t}\, \frac{c_t^{1-p}}{1-p}\, \mathrm{d}t\right]$
- HJB equation:

$$\begin{cases} \delta v = \max_{\pi\in\mathbb{R}}\left[\mu\pi v_x + \frac{1}{2}\,\sigma^2\pi^2 v_{xx}\right] + \max_{\alpha z\le c\le z}\left[\frac{c^{1-p}}{1-p} - c v_x\right] \\ v(0,z) = 0 \\ v_z(w^* z, z) = 0 = v_{xz}(w^* z, z) \end{cases}$$

- The last conditions are the *“smooth-pasting”* and *“super-contact”* conditions
  See, for example, Dixit (1991) and Dumas (1991)

- $V(x,z)$ is homogeneous of degree $1-p$ with respect to x and z

$$V(\beta x, \beta z) = \beta^{1-p} V(x,z); \quad \beta > 0$$

- Using the ansatz $V(x,z) = z^{1-p} U(x/z)$ leads to

$$\begin{cases} \delta U = \max_{\hat{\pi} \in \mathbb{R}} \left[\mu \hat{\pi} U_w + \frac{1}{2} \sigma^2 \hat{\pi}^2 U_{ww} \right] + \max_{\alpha \le \hat{c} \le 1} \left[\frac{\hat{c}^{1-p}}{1-p} - \hat{c} U_w \right] \\ U(0) = 0 \\ (1-p) U(w^*) - w^* U_w(w^*) = 0 \\ p U_w(w^*) + w^* U_{ww}(w^*) = 0 \end{cases}$$

- Once we obtained $\hat{\pi}^*$ and $\hat{c}^*$, we get π^* and c^* via $\pi^*(x,z) = \hat{\pi}^*(x/z) z$ and $c^*(x,z) = \hat{c}^*(x/z) z$

- Assuming U is increasing and concave with respect to w

$$\frac{1}{\kappa}\frac{U_w^2}{U_{ww}} + \delta U = \begin{cases} \dfrac{\alpha^{1-p}}{1-p} - \alpha U_w, & 0 \le w \le w_\alpha \\ \dfrac{p}{1-p}\big(U_w(w)\big)^{-\frac{1-p}{p}}, & w_\alpha < w < w_1 \\ \dfrac{1}{1-p} - U_w, & w_1 \le w \le w^* \end{cases}$$

- Here, $\kappa := \frac{2\sigma^2}{\mu^2}$ and w_α and w_1 are free boundaries satisfying

$$U_w(w_\alpha) = \alpha^{-p} \text{ and } U_w(w_1) = 1$$

- We can linearize the equation by applying the Legendre transform

- Define $y_0 := U_w(0) \geq \alpha^{-p}$, $y^* := U_w(w^*) \leq 1$, and

$$\widehat{U}(y) := \sup_{0<w<w^*} \{U(w) - wy\}; \quad y^* \leq y \leq y_0$$

- $\widehat{U}$ satisfies:

$$y^2 \widehat{U}_{yy} + \kappa\delta y \widehat{U}_y - \kappa\delta \widehat{U} = \begin{cases} \kappa\left(\alpha y - \frac{\alpha^{1-p}}{1-p}\right), & \alpha^{-p} \leq y \leq y_0 \\ -\frac{\kappa p}{1-p} y^{-\frac{1-p}{p}}, & 1 < y < \alpha^{-p} \\ \kappa\left(y - \frac{1}{1-p}\right), & y^* \leq y \leq 1 \end{cases}$$

$$\widehat{U}(y_0) = 0 = \widehat{U}_y(y_0)$$

$$(1-p)\widehat{U}(y^*) + py^* \widehat{U}_y(y^*) = 0$$

$$\widehat{U}_y(y^*) + py^* \widehat{U}_{yy}(y^*) = 0$$

Lemma (y_0 and y^*)

There exist unique constants $\eta^* = y_0\alpha^p > 1$ *and* $0 < y^* < 1$ *that solves the system:*

$$
\begin{cases}
\ln \dfrac{\eta^\alpha}{y} + \dfrac{\alpha}{\eta(1-p)} - \dfrac{1}{y} = \alpha(1+p) - 1 \\[2ex]
\alpha^{1-p(1+\kappa\delta)} \big(p(1+\kappa\delta) - 1\big) \left(\dfrac{\kappa}{1+\kappa\delta}\, \eta^{1+\kappa\delta} - \dfrac{1}{\delta(1-p)}\, \eta^{\kappa\delta} \right) \\[2ex]
\qquad + \left(\dfrac{\kappa}{1+\kappa\delta}\, y^{1+\kappa\delta} - \dfrac{1}{\delta}\, y^{\kappa\delta} \right) = \dfrac{\alpha^{1-p(1+\kappa\delta)} - 1}{\delta(1+\kappa\delta)}
\end{cases}
$$

Proposition ($\widehat{U}$)

$\widehat{U}$ is given by

$$\widehat{U}(y) = \begin{cases} C_1 y + C_2 y^{-\kappa\delta} + \frac{\kappa\alpha}{1+\kappa\delta}\, y \ln y + \frac{\alpha^{1-p}}{\delta(1-p)}\,, & \alpha^{-p} \leq y \leq y_0 \\ C_3 y + C_4 y^{-\kappa\delta} + \frac{\kappa}{1-p}\, \frac{p^3}{p(1+\kappa\delta)-1}\, y^{-\frac{1-p}{p}} & 1 < y < \alpha^{-p} \\ C_5 y + C_6 y^{-\kappa\delta} + \frac{\kappa}{1+\kappa\delta}\, y \ln y + \frac{1}{\delta(1-p)}\,, & y^* \leq y \leq 1 \end{cases}$$

with $C_1, \ldots, C_6$ given in the next slide.

Moreover, $\widehat{U}$ is strictly decreasing and strictly convex with continuous second derivative on (y^*, y_0).

Solution: $\widehat{U}$

$$C_1 = -\frac{\kappa\alpha}{1+\kappa\delta}\left(\ln\eta^* - p\ln\alpha + \frac{1}{\eta^*(1-p)} + \frac{1}{1+\kappa\delta}\right)$$

$$C_2 = \frac{\alpha^{1-p(1+\kappa\delta)}}{1+\kappa\delta}\left(\frac{\kappa}{1+\kappa\delta}(\eta^*)^{1+\kappa\delta} - \frac{1}{\delta(1-p)}(\eta^*)^{\kappa\delta}\right) > 0$$

$$C_3 = -\frac{\kappa\alpha}{1+\kappa\delta}\left(\ln\eta^* + \frac{1}{\eta^*(1-p)} - (1+p)\right)$$

$$C_4 = \frac{\alpha^{1-p(1+\kappa\delta)}}{1+\kappa\delta}\left(\frac{\kappa}{1+\kappa\delta}(\eta^*)^{1+\kappa\delta} - \frac{1}{\delta(1-p)}(\eta^*)^{\kappa\delta} - \frac{1}{\delta(1+\kappa\delta)\big(p(1+\kappa\delta)-1\big)}\right) < 0$$

$$C_5 = -\frac{\kappa}{1+\kappa\delta}\left(\alpha\ln\eta^* + \frac{\alpha}{\eta^*(1-p)} + (1-\alpha)(1+p) + \frac{1}{1+\kappa\delta}\right)$$

$$C_6 = \frac{\alpha^{1-p(1+\kappa\delta)}}{1+\kappa\delta}\left(\frac{\kappa}{1+\kappa\delta}(\eta^*)^{1+\kappa\delta} - \frac{1}{\delta(1-p)}(\eta^*)^{\kappa\delta}\right) - \frac{\alpha^{1-p(1+\kappa\delta)} - 1}{\delta(1+\kappa\delta)^2\big(p(1+\kappa\delta)-1\big)} > 0$$

- We find $U(w)$ by reversing the Legendre transform

$$U(w) = \widehat{U}(y) - y\widehat{U}_y(y), \text{ where } y \in [y^*, y_0] \text{ uniquely solves } \widehat{U}_y(y) = -w$$

- We then find the (candidate) value function $V(x, z) = z^{1-p}U(x/z)$

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- The critical values w_α, w_1, and w^* are given by

$$w_\alpha = \frac{\kappa\alpha}{1+\kappa\delta}\left\{\ln\eta^* + \left(\frac{\kappa\delta}{1+\kappa\delta} - \frac{1}{\eta^*(1-p)}\right)\left((\eta^*)^{1+\kappa\delta} - 1\right)\right\}$$

$$w_1 = \frac{\kappa}{1+\kappa\delta}\left\{\ln y^* + p + \left(\frac{1}{y^*} - \frac{\kappa\delta}{1+\kappa\delta}\right)\left(1 + \frac{(y^*)^{1+\kappa\delta}}{p(1+\kappa\delta)-1}\right)\right\}$$

$$w^* = \frac{\kappa p}{p(1+\kappa\delta)-1}\left\{\frac{1}{y^*} - (1-p)\right\}$$

- For $0 \leq x \leq w^* z$: Let $y \in [y^*, y_0]$ be the unique solution of $\widehat{U}_y(y) = -x/z$

$$\pi^*(x,z) = -\frac{\mu}{\sigma^2}\frac{zU_w(x/z)}{U_{ww}(x/z)} = \frac{\mu}{\sigma^2}\, zy\widehat{U}_{yy}(y)$$

$$c^*(x,z) = \begin{cases} \alpha z, & 0 \leq x \leq w_\alpha z, \\ y^{-\frac{1}{p}}\, z, & w_\alpha z < x < w_1 z, \\ z, & w_1 z \leq x < w^* z. \end{cases}$$

- For $x > w^* z$:

$$\pi^*(x,z) = -\frac{\mu}{\sigma^2}\frac{U_w(w^*)}{w^* U_{ww}(w^*)}\, x = \frac{\mu}{\sigma^2}\frac{y^*}{w^*}\,\widehat{U}_{yy}(y^*)\, x$$

$$c^*(x,z) = \frac{x}{w^*}$$

- To verify the solution, we need to show that, for all $x, z \geq 0$, $\pi \in \mathbb{R}$, and $c \geq \alpha z$

(i) $v_z(x,z) \leq 0$

(ii) $\dfrac{1}{2}\,\sigma^2\pi^2 v_{xx}(x,z) + (\mu\pi - c)v_x(x,z) - \delta v(x,z) + \dfrac{c^{1-p}}{1-p} \leq 0$

(iii) the *"transversality condition:"* $\liminf_{n\to\infty} \mathbb{E}^x\left(\mathrm{e}^{-\delta\tau_n} v(X_{\tau_n}, z_{\tau_n})\right) = 0$

$\{\tau_n\}_{n=1}^{\infty}$ is a sequence of bounded stopping times satisfying $\tau_n \to \infty$ $\mathbb{P}$-a.s.

(iv) $\max\left[v_z,\ \tfrac{1}{2}\,\sigma^2(\pi^*)^2 v_{xx} + (\mu\pi^* - c^*)v_x + \tfrac{(c^*)^{1-p}}{1-p} - \delta v\right] = 0; \quad x, z > 0$

(v) The following SDE has a unique strong solution

$$\begin{cases} \mathrm{d}X_t^* = \Big(\mu\pi^*(X_t^*, z_t^*) - c^*(X_t^*, z_t^*)\Big)\mathrm{d}t + \sigma\pi^*(X_t^*, z_t^*)\,\mathrm{d}W_t; \quad t \geq 0, \\ z_t^* = \max\left\{z,\ \sup_{0\leq s<t} c^*(X_s^*, z_s^*)\right\}; \quad t \geq 0, \\ X_0^* = x, \end{cases}$$

and $\Big(\pi^*(X_t^*, z_t^*), c^*(X_t^*, z_t^*)\Big)$ is admissible

Thank you for your attention!

Angoshtari, B., E. Bayraktar, and V. Young: Optimal dividend distribution under drawdown and ratcheting constraints on dividend rates (2018). Preprint, Available at arXiv:1806.07499.

Albrecher, H., N. Bäuerle, and M. Bladt: Dividends: From refracting to ratcheting (2018). Working Paper, University of Lausanne.

Arun, T.: The Merton problem with a drawdown constraint on consumption (2012). Working Paper, University of Cambridge, Available at: https://arxiv.org/abs/1210.5205.

Asmussen, S. and M. Taksar: Controlled diffusion models for optimal dividend pay-out. *Insurance: Mathematics and Economics*, volume 20, no. 1: pp. 1–15 (1997).

Avanzi, B.: Strategies for dividend distribution: A review. *North American Actuarial Journal*, volume 13, no. 2: pp. 217–251 (2009).

Cvitanić, J. and I. Karatzas: On portfolio optimization under "drawdown" constraints. *IMA Lecture Notes in Mathematical Applications*, volume 65: pp. 77–88 (1995).

De Finetti, B.: Su un'Impostazione alternativa della teoria collettiva del rischio. *Transactions of the XVth International Congress of Actuaries*, volume 2: pp. 433–443 (1957).

Dixit, A. K.: A simplified treatment of the theory of optimal regulation of Brownian motion. *Journal of Economic Dynamics and Control*, volume 15, no. 4: pp. 657–673 (1991).

Dumas, B.: Super contact and related optimality conditions. *Journal of Economic Dynamics and Control*, volume 15, no. 4: pp. 675–685 (1991).

Dybvig, P. H.: Dusenberry's racheting of consumption: Optimal dynamic consumption and investment given intolerance for any decline in standard of living. *Review of Economic Studies*, volume 62, no. 2: pp. 287–313 (1995).

Elie, R. and N. Touzi: Optimal lifetime consumption and investment under a drawdown constraint. *Finance and Stochastics*, volume 12, no. 3: pp. 299–330 (2008).

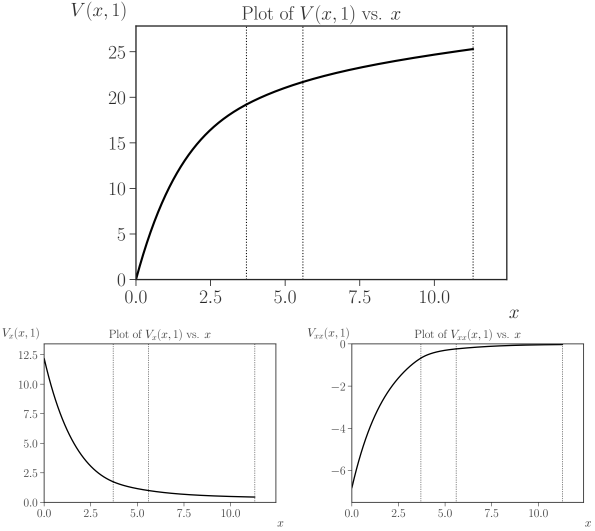
Gerber, H. U. and E. S. W. Shiu: On optimal dividends: From reflection to refraction. *Journal of Computational and Applied Mathematics*, volume 186, no. 1: pp. 4–22 (2006).

Grossman, S. J. and Z. Zhou: Optimal investment strategies for controlling drawdowns. *Mathematical Finance*, volume 3, no. 3: pp. 241–276 (1993).

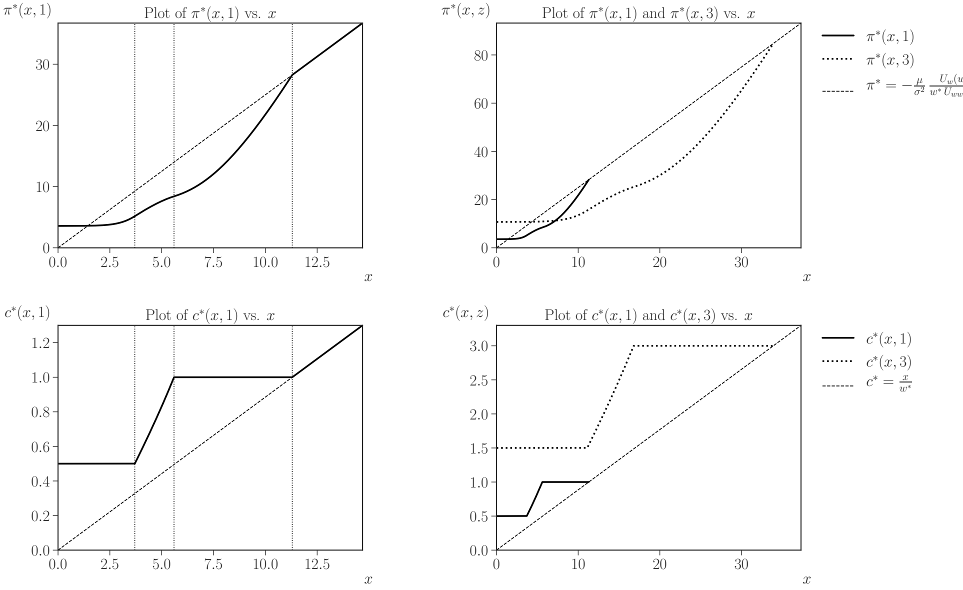
Merton, R. C.: Lifetime portfolio selection under uncertainty: The continuous-time case. *Review of Economics and Statistics*, volume 51, no. 3: pp. 247–257 (1969).

Plot of the Value Function

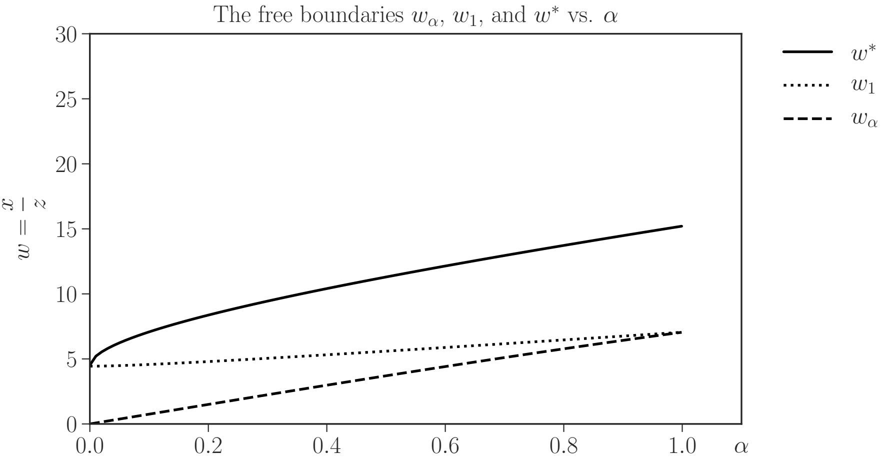
$\mu = 0.08$, $\sigma = 0.2$, $\delta = 0.2$, $\alpha = 0.5$, $p = 0.8$, $w_\alpha = 3.703$, $w_1 = 5.5947$, and $w^* = 11.2992$



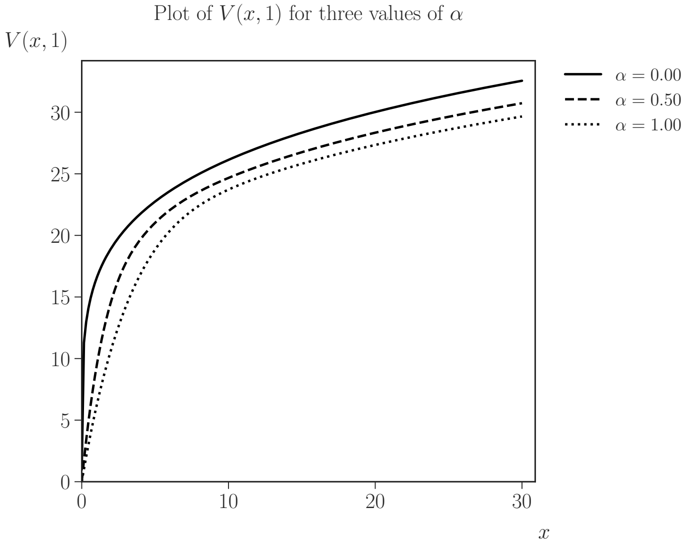
Plot of the Optimal Policy



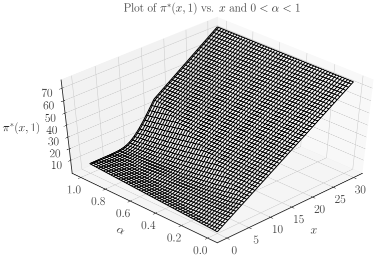
Sensitivity of the Free Boundaries w.r.t. α

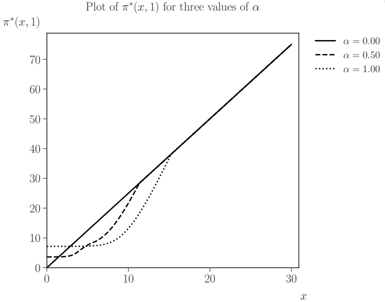


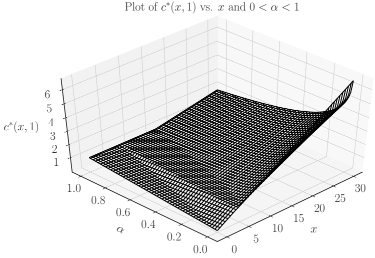
Sensitivity of the Value function w.r.t. α

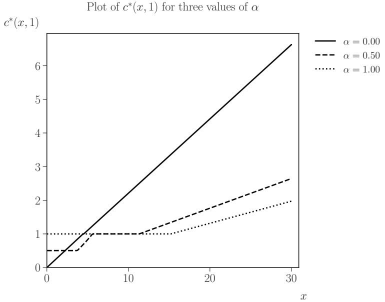


Sensitivity of the Optimal Policy w.r.t. α









Sensitivity of the Value Function w.r.t. p