Imperfect Competition Among Liquidity Providers

Angad Singh

Caltech

November 2018

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Motivation

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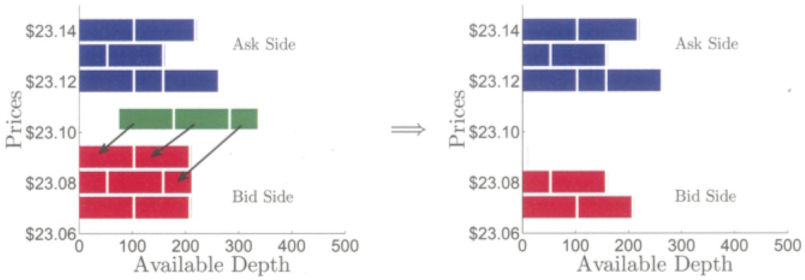

$23.14
$23.12
$23.10
$23.08
$23.06
Prices
Ask Side
Bid Side
0
100
200
300
400
500
Available Depth
⟹
$23.14
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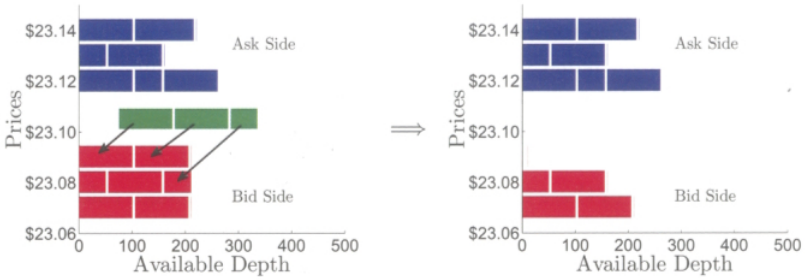

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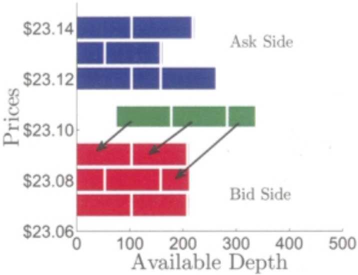

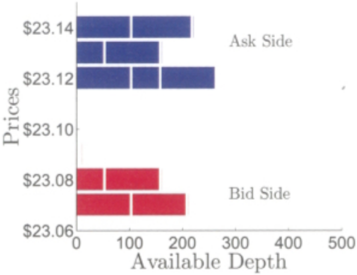

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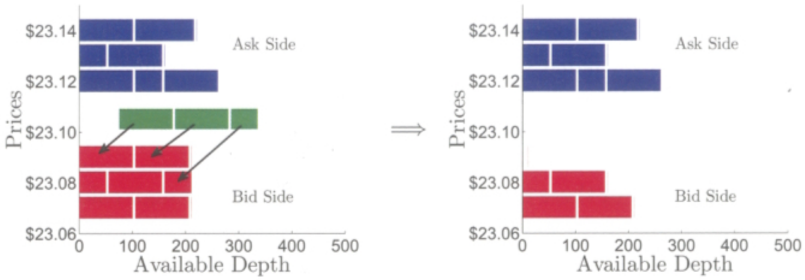

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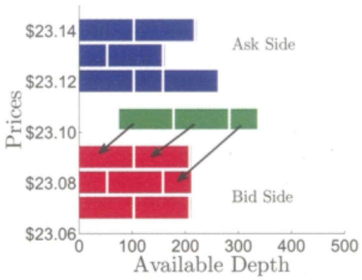

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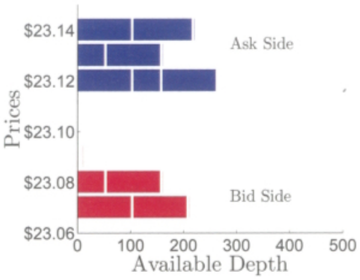

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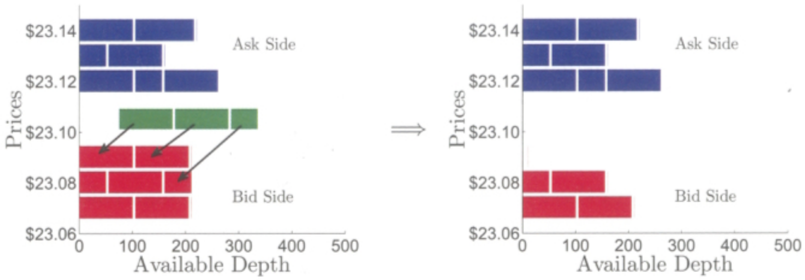

- LO specifies a price and a quantity → **price elastic demand**
- MO specifies only a quantity → **price inelastic demand**
- Trade means MO executes against LO

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- Smooth trading on an infinite horizon
- Two types of traders:
  - N Market Makers (MMs)
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- Trades occur at a uniform price p_t to be determined endogenously

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- LTs inventory is $-S_t$, where

$$\begin{aligned} dS_t &= F_t dt \\ dF_t &= -\psi F_t dt + \sigma_F dB_t^F, \qquad \psi > 0 \end{aligned}$$

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- MMs cash evolves according to

$$dC_t^n = -q_t^n p_t dt$$

MM Objectives

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- MMs valuation of the stock is exogenously given as D_t where

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$$\mathbb{E}\Big[\int_0^\infty e^{-\rho t}\Big(dW_t^n - \frac{\gamma}{2} d\langle W^n\rangle_t\Big)\Big] =$$
$$\mathbb{E}\Big[\int_0^\infty e^{-\rho t}\Big(-q_t^n(p_t - D_t) + \mu X_t^n - \frac{\gamma\sigma_D^2}{2}(X_t^n)^2\Big)dt\Big]$$

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- Price and trades are set in Nash equilibrium

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- A profile is **linear** and **symmetric** if $\exists\ a < 0, e > 0$, and $b, c, \xi \in \mathbb{R}$ s.t.

$$\alpha_t^n = aX_t^n + bD_t + cS_t + \xi$$
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- Will focus on linear symmetric Nash equilibrium

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Explicit Equilibrium

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If $N \geq 3$ then there is a unique linear symmetric Nash equilibrium. In equilibrium the price is

$$p_t = D_t + \frac{\mu}{\rho} - \frac{1}{\rho}\frac{\gamma}{N}\sigma_D^2 S_t - \frac{\gamma}{N}\frac{N-1}{N-2}\frac{\sigma_D^2}{\rho+\psi}\left(\frac{1}{\delta}+\frac{1}{\rho}\right)F_t$$

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where

$$\kappa = \rho(N-2)\frac{\rho+\psi}{\rho+\delta}$$

$$\delta = \sqrt{\rho^2 + 2\rho(\rho+\psi)(N-2)}$$

Sketch of Proof

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- On equilibrium

$$\hat{p}_t = \frac{a}{N} S_t + bD_t + cS_t + \xi - \frac{e}{N} F_t$$

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- Off equilibrium

$$p_t = \frac{a}{N-1}(S_t - X_t) + bD_t + cS_t + \xi - \frac{e}{N-1}(F_t - q_t)$$
$$p_t = \alpha_t - \beta_t q_t$$

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&= M_t + \Lambda X_t + \lambda q_t
\end{aligned}
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- Dynamic Programming, HJB, Feedback Controller

$$\frac{2e}{N-1}\hat{q} = V_x - \left[\left(\frac{a}{N-1} + c\right)s - \frac{a}{N-1}x + (b-1)d + \xi - \frac{e}{N-1}f\right]$$

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Interpreting the Price

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$$p_t = D_t + \frac{\mu}{\rho} - \frac{1}{\rho}\frac{\gamma}{N}\sigma_D^2 S_t - \frac{\gamma}{N}\frac{N-1}{N-2}\frac{\sigma_D^2}{\rho+\psi}\left(\frac{1}{\delta}+\frac{1}{\rho}\right)F_t$$

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- First term is MMs current asset valuation
- Second term is a premium for expected valuation growth

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- First term is MMs current asset valuation
- Second term is a premium for expected valuation growth
- Third term is a discount to compensate MMs for bearing risk

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- Fourth term captures liquidity

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Definition

$Price\ Impact := \frac{\gamma}{N}\frac{N-1}{N-2}\frac{\sigma_D^2}{\rho+\psi}\left(\frac{1}{\delta}+\frac{1}{\rho}\right)$

$Liquidity := \frac{1}{Price\ \ Impact}$

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   Liquidity is increasing in market maker competition.

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4. $\frac{\partial}{\partial \psi} Liquidity > 0.$
   Liquidity is decreasing in order flow risk.

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- *Price Impact* $\rightarrow 0$ as $\psi \rightarrow \infty$

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- *Price Impact* $\to \frac{\gamma_0}{\rho} \frac{\sigma_D^2}{\rho + \psi}$ as $N \to \infty$ with $\frac{\gamma}{N} = \gamma_0$ fixed
- In both cases the price is asymptotic to

$$D_t + \frac{\mu}{\rho} - \mathbb{E}_t \left[\int_t^\infty e^{-\rho(T-t)} \gamma_0 \sigma_D^2 S_T dT \right]$$

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Definition

Rate of Convergence to Efficiency $:= \kappa := \rho(N-2)\frac{\rho+\psi}{\rho+\delta}$

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- Stochastic differential game to compete for order flow

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  - Imperfect competition
  - Order flow risk
- Risk discount today is the present value of future risk discounts
- Risk reallocation among liquidity providers can be slow