Asset price dynamics in stock-flow consistent macroeconomic model

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James Tobin's contributions to economics

- Tobin received the 1981 Nobel Memorial Prize "for his analysis of financial markets and their relations to expenditure decisions, employment, production and prices".
- Well-known contributions included: foundations of modern portfolio theory (with Markowitz), in particular the Separation Theorem (1958), life-cycle model of consumption, Tobit estimator, Tobin's q, Tobin's tax, ...
- Key forgotten contribution: financial intermediation, portfolio balances, flow of funds models and the credit channel.

Tobin 1969: A General Equilibrium Approach to Monetary Theory

- Specification of (i) a menu of assets, (ii) the factors that determine the demands and supplies of the various assets, and (iii) the manner in which asset prices and interest rates clear these interrelated markets.
- Spending decisions are independent from portfolio decisions.
- Each asset i has a rate of return r_i and each sector j has a net demand f_{ij} for asset i.
- Adding up constraint: for each rate of return r_k,

$$\sum_{i=1}^{n} \frac{\partial f_{ij}}{\partial r_k} = 0.$$

- Paper proceeds to analyze several special cases: money-capital, money-treasuries-capital, banks, etc.
- Victim of the Microfoundations Revolution.

SMD theorem: something is rotten in GE land

Stock-Flow Consistent models

- Stock-flow consistent models emerged in the last decade as a common language for many heterodox schools of thought in economics.
- They consider both real and monetary factors simultaneously.
- Specify the balance sheet and transactions between sectors.
- Accommodate a number of behavioural assumptions in a way that is consistent with the underlying accounting structure.
- Reject the RARE individual (representative agent with rational expectations) in favour of SAFE (sectoral average with flexible expectations) modelling.
- See Godley and Lavoie (2007) for the full framework.

Balance Sheets

Balance Sheet	Households	Firms		Banks	Central Bank	Government	Sum
		current	capital				
Cash	$+H_h$			$+H_b$	$-H$		0
Deposits	$+M_h$		$+M_f$	$-M$			0
Loans			$-L$	$+L$			0
Bills	$+B_h$			$+B_b$	$+B_c$	$-B$	0
Equities	$+p_f E_f + p_b E_b$		$-p_f E_f$	$-p_b E_b$			0
Advances				$-A$	$+A$		0
Capital			$+pK$				pK
Inventory			$+cV$				cV
Sum (net worth)	X_h	0	X_f	X_b	0	$-B$	X

Table: Balance sheet in an example of a general SFC model.

Transactions

Transactions	Households	Firms: current	Firms: capital	Banks	Central Bank	Government	Sum
Consumption	$-pC_h$	$+pC$		$-pC_b$			0
Investment		$+pI_k$	$-pI_k$				0
Change in Inventory		$+c\dot{V}$	$-c\dot{V}$				0
Gov spending		$+pG$				$-pG$	0
Acct memo [GDP]		$[pY]$					
Wages	$+W$	$-W$					0
Taxes	$-T_h$	$-T_f$				$+T$	0
Interest on deposits	$+r_M.M_h$	$+r_M.M_f$		$-r_M.M$			0
Interest on loans		$-r_L.L$		$+r_L.L$			0
Interest on bills	$+r_B.B_h$			$+r_B.B_b$	$+r_B.B_c$	$-r_B.B$	0
Profits	$+\Pi_d+\Pi_b$	$-\Pi$	$+\Pi_u$	$-\Pi_b$	$-\Pi_c$	$+\Pi_c$	0
Sum	S_h	0	$S_f-pI_k-c\dot{V}$	S_b	0	S_g	0

Table: Transactions in an example of a general SFC model.

Flow of Funds

Flow of Funds	Households	Firms current	Firms capital	Banks	Central Bank	Government	Sum
Cash	$+\dot{H}_h$			$+\dot{H}_b$	$-\dot{H}$		0
Deposits	$+\dot{M}_h$		$+\dot{M}_f$	$-\dot{M}$			0
Loans			$-\dot{L}$	$+\dot{L}$			0
Bills	$+\dot{B}_h$			$+\dot{B}_b$	$+\dot{B}_c$	$-\dot{B}$	0
Equities	$+p_f\dot{E}_f + p_b\dot{E}_b$		$-p_f\dot{E}_f$	$-p_b\dot{E}_b$			0
Advances				$-\dot{A}$	$+\dot{A}$		0
Capital			$+pI$				pI
Sum	S_h	0	S_f	S_b	0	S_g	pI
Change in Net Worth	$(S_h + \dot{p}_f E_f + \dot{p}_b E_b)$	$(S_f - \dot{p}_f E_f + \dot{p}K - p\delta K)$		$(S_b - \dot{p}_b E_b)$		S_g	$\dot{p}K + p\dot{K}$

Table: Flow of funds in an example of a general SFC model.

Example: household balance sheet US 2013

B.100 Balance Sheet of Households and Nonprofit Organizations (1)
Billions of dollars; amounts outstanding end of period, not seasonally adjusted

			2010	2011	2012	2013
1	**FL152000005**	**Assets**	**77130.1**	**78258.0**	**84441.4**	**94042.3**
2	FL152010005	Nonfinancial assets	23323.3	23265.8	25007.3	27544.4
3	FL155035005	Real estate	18330.9	18111.2	19711.8	22069.7
4	FL155035015	Households (2,3)	16347.4	15939.7	17394.5	19407.5
5	FL165035005	Nonprofit organizations	1983.6	2171.5	2317.2	2662.2
6	FL165015205	Equipment (nonprofits) (4)	290.6	304.6	315.1	323.7
7	FL165013765	Intellectual property products (nonprofits) (4)	115.0	123.6	132.4	140.0
8	FL155111005	Consumer durable goods (4)	4586.7	4726.4	4848.0	5011.0
9	FL154090005	Financial assets	53806.9	54992.2	59434.1	66497.9
10	FL154000025	Deposits	8059.4	8736.8	9241.5	9572.3
11	FL153091003	Foreign deposits	49.7	46.9	45.1	48.4
12	FL153020005	Checkable deposits and currency	423.6	752.0	897.8	1004.7
13	FL153030005	Time and savings deposits	6455.9	6827.7	7191.2	7388.7
14	FL153034005	Money market fund shares	1130.2	1110.2	1107.4	1130.4
15	FL154004005	Credit market instruments	5834.0	5425.5	5422.2	5446.0
16	FL163069103	Open market paper	21.1	19.4	18.8	15.0
17	FL153061505	Treasury securities	1134.4	715.6	941.0	935.4
18	FL153061705	Agency- and GSE-backed securities	353.7	304.6	134.2	123.9
19	FL153062005	Municipal securities	1871.8	1808.3	1665.8	1626.3
20	FL153063005	Corporate and foreign bonds	2248.3	2379.0	2468.8	2578.0
21	FL153069803	Other loans and advances (5)	26.2	23.4	20.9	25.9
22	FL153065005	Mortgages	100.1	100.8	86.9	80.4
23	FL163066223	Consumer credit (student loans)	78.4	74.5	65.6	59.1
24	FL153064105	Corporate equities (2)	9995.3	9025.4	10412.8	13309.6
25	FL153064205	Mutual fund shares (6)	4600.2	4502.9	5408.7	6890.1
26	FL153067005	Security credit	725.2	728.1	757.8	813.3
27	FL153040005	Life insurance reserves	1137.3	1203.6	1186.1	1242.2
28	FL153050005	Pension entitlements (7)	16751.6	17128.1	18093.8	19563.8
29	FL152090205	Equity in noncorporate business (8)	6895.6	7366.9	8038.4	8760.8
30	FL153090005	Miscellaneous assets	808.2	878.8	873.6	897.6
31	**FL154190005**	**Liabilities**	**13766.5**	**13566.0**	**13626.8**	**13768.2**
32	FL154104005	Credit market instruments	13214.8	13052.9	13044.2	13146.1
33	FL153165105	Home mortgages (9)	9912.7	9697.5	9481.3	9386.2
34	FL153166000	Consumer credit	2647.4	2755.9	2923.6	3097.4
35	FL163162003	Municipal securities (10)	263.2	255.5	241.8	227.8
36	FL153168005	Depository institution loans n.e.c.	61.0	11.5	62.6	92.7
37	FL153169005	Other loans and advances	136.1	138.1	139.3	141.3
38	FL163165505	Commercial mortgages (10)	194.3	194.3	195.9	200.8
39	FL153167005	Security credit	278.2	238.9	303.7	339.2
40	FL163170003	Trade payables (10)	248.8	250.0	254.8	255.0
41	FL543077073	Deferred and unpaid life insurance premiums	24.7	24.3	24.9	27.9
42	**FL152090005**	**Net worth**	**63363.7**	**64692.0**	**70814.6**	**80274.1**

Example: NIPA US 2012

Table A. Summary National Income and Product Accounts, 2012

[Billions of dollars]

Account 1. Domestic Income and Product Account

Line		
1	Compensation of employees, paid	8,620.2
2	Wages and salaries	6,935.1
3	Domestic (3-12)	6,920.5
4	Rest of the world (5-11)	14.6
5	Supplements to wages and salaries (3-14)	1,684.9
6	Taxes on production and imports (4-15)	1,128.9
7	Less: Subsidies (4-8)	57.5
8	Net operating surplus	4,033.2
9	Private enterprises (2-19)	4,060.9
10	Current surplus of government enterprises (4-25)	-27.7
11	Consumption of fixed capital (6-14)	2,540.9
12	**Gross domestic income**	**16,261.8**
13	Statistical discrepancy (6-20)	-17.0
14	**GROSS DOMESTIC PRODUCT**	**16,244.8**

Line		
15	Personal consumption expenditures (3-3)	11,149.6
16	Goods	3,769.7
17	Durable goods	1,202.7
18	Nondurable goods	2,567.0
19	Services	7,379.8
20	Gross private domestic investment	2,475.2
21	Fixed investment (6-2)	2,409.1
22	Nonresidential	1,970.0
23	Structures	437.3
24	Equipment	907.5
25	Intellectual property products	625.5
26	Residential	439.2
27	Change in private inventories (6-4)	66.1
28	Net exports of goods and services	-547.2
29	Exports (5-1)	2,195.9
30	Imports (5-9)	2,743.1
31	Government consumption expenditures and gross investment (4-1 plus 6-3)	3,167.0
32	Federal	1,295.7
33	National defense	817.1
34	Nondefense	478.6
35	State and local	1,871.3
36	**GROSS DOMESTIC PRODUCT**	**16,244.8**

Account 2. Private Enterprise Income Account

Line		
1	Income payments on assets	2,684.2
2	Interest and miscellaneous payments (3-21 and 3-20 and 4-20 and 5-13)	2,437.2
3	Dividend payments to the rest of the world (5-14)	141.1
4	Reinvested earnings on foreign direct investment in the United States (5-15)	105.9
5	Business current transfer payments (net)	108.9
6	To persons (net) (3-24)	41.4
7	To government (net) (4-23)	76.6
8	To the rest of the world (net) (5-19)	-9.1
9	Proprietors' income with IVA and CCAdj (3-17)	1,224.9
10	Rental income of persons with CCAdj (3-18)	541.2
11	Corporate profits with IVA and CCAdj	2,028.5
12	Taxes on corporate income	434.8
13	To government (4-16)	402.4
14	To the rest of the world (5-19)	32.4
15	Profits after tax with IVA and CCAdj	1,574.7
16	Net dividends (3-21 plus 4-21)	770.3
17	Undistributed corporate profits with IVA and CCAdj (6-12)	804.3
18	**USES OF PRIVATE ENTERPRISE INCOME**	**6,536.7**

Line		
19	Net operating surplus, private enterprises (1-9)	4,060.9
20	Income receipts on assets	2,475.8
21	Interest (2-2 and 3-4 and 4-7 and 5-6)	1,809.8
22	Dividend receipts from the rest of the world (5-6)	297.8
23	Reinvested earnings on U.S. direct investment abroad (5-7)	368.1
24	**SOURCES OF PRIVATE ENTERPRISE INCOME**	**6,536.7**

Account 3. Personal Income and Outlay Account

Line		
1	Personal current taxes (4-14)	1,498.0
2	Personal outlays	11,558.4
3	Personal consumption expenditures (1-15)	11,149.6
4	Personal interest payments (2-21 and 3-20 and 4-20 and 5-13)	248.4
5	Personal current transfer payments	160.4
6	To government (4-24)	88.5
7	To the rest of the world (net) (5-17)	71.9
8	Personal saving (6-11)	687.4
9	**PERSONAL TAXES, OUTLAYS, AND SAVING**	**13,743.8**

Line		
10	Compensation of employees, received	8,611.6
11	Wages and salaries	6,926.8
12	Domestic (1-3)	6,920.5
13	Rest of the world (5-3)	6.3
14	Supplements to wages and salaries (1-5)	1,684.8
15	Employer contributions for employee pension and insurance funds	1,170.6
16	Employer contributions for government social insurance	514.2
17	Proprietors' income with IVA and CCAdj (2-9)	1,224.9
18	Rental income of persons with CCAdj (2-10)	541.2
19	Personal income receipts on assets	1,958.5
20	Personal interest income (2-2 plus 3-4 plus 4-7 plus 5-6 less 2-21 less 4-20 less 5-13)	1,211.6
21	Personal dividend income (2-16 less 4-21)	746.8
22	Personal current transfer receipts	2,358.3
23	Government social benefits (4-4)	2,316.9
24	From business (net) (2-6)	41.4
25	Less: Contributions for government social insurance, domestic (4-18)	953.7
26	**PERSONAL INCOME**	**13,743.8**

Goodwin Model - SFC matrix

Balance Sheet	Households	Firms		Sum
		current	capital	
Capital			$+pK$	pK
Sum (net worth)	0	0	V_f	pK
Transactions				
Consumption	$-pC$	$+pC$		0
Investment		$+pI$	$-pI$	0
Acct memo [GDP]		$[pY]$		
Wages	$+W$	$-W$		0
Profits		$-\Pi$	$+\Pi_u$	0
Sum	0	0	0	0
Flow of Funds				
Capital			$+pI$	pI
Sum	0	0	Π_u	pI
Change in Net Worth	0	$pI + \dot{p}K - p\delta K$		$\dot{p}K + p\dot{K}$

Table: SFC table for the Goodwin model.

Goodwin Model - Differential equations

- Define

$$\omega = \frac{\mathrm{w}\ell}{pY} = \frac{\mathrm{w}}{pa} \quad \text{(wage share)}$$

$$\lambda = \frac{\ell}{N} = \frac{Y}{aN} \quad \text{(employment rate)}$$

- It then follows that

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} - \frac{\dot{p}}{p} - \frac{\dot{a}}{a} = \Phi(\lambda, i, i^e) - i - \alpha$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{1-\omega}{\nu} - \alpha - \beta - \delta$$

- In the original model, all quantities were real (i.e divided by p), which is equivalent to setting $i = i^e = 0$.

Where does Φ come from?

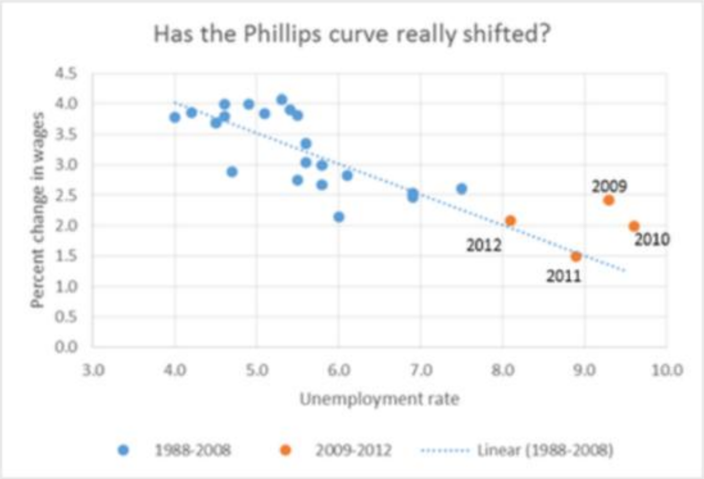
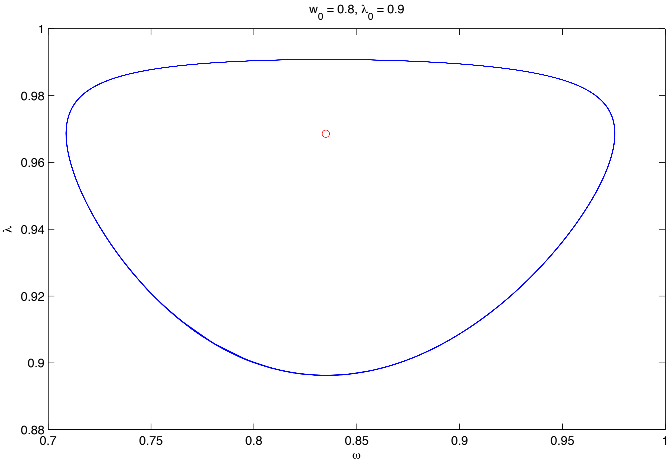


Figure: Krugman - July 15, 2014

Example 1: Goodwin model



Testing Goodwin on OECD countries

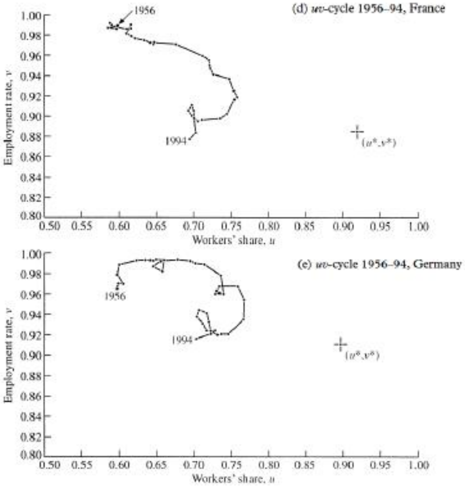


Figure: Harvie (2000)

Correcting Harvie (1970 to 2009)

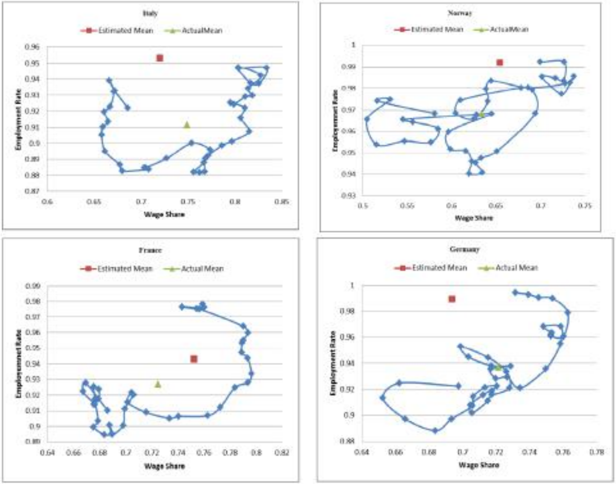


Figure: Grasselli and Maheshwari (2014, in progress)

What about shocks?

- Nguyen Huu and Costa Lima (2014) introduce stochastic productivity of the form

$$da_t := a_t d\alpha_t = a_t[\alpha dt - \sigma(\lambda_t)dW_t]$$

leading to a modified model of the form

$$\frac{\dot{\omega}}{\omega} = \Phi(\lambda) - \alpha + \sigma^2(\lambda_t)dt + \sigma(\lambda_t)dW_t$$
$$\frac{\dot{\lambda}}{\lambda} = \frac{1-\omega}{\nu} - \alpha - \beta - \delta + \sigma^2(\lambda_t)dt + \sigma(\lambda_t)dW_t$$

- They then prove the existence of stochastic orbits generalizing the original Goodwin cycles.

Stochastic orbits of a Goodwin model with productivity shocks

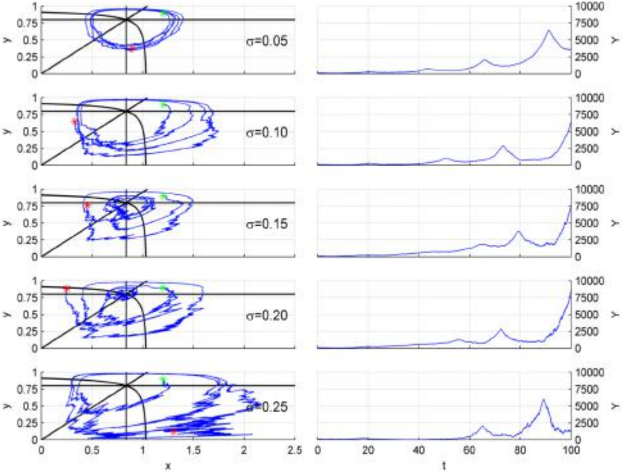


Figure: Figure 3 in Nguyen Huu and Costa Lima (2014)

SFC table for Keen (1995) model

Balance Sheet	Households	Firms current	Firms capital	Banks	Sum
Deposits	$+D$			$-D$	0
Loans			$-L$	$+L$	0
Capital			$+pK$		pK
Sum (net worth)	V_h	0	V_f	0	pK
Transactions					
Consumption	$-pC$	$+pC$			0
Investment		$+pI$	$-pI$		0
Acct memo [GDP]		$[pY]$			
Wages	$+W$	$-W$			0
Interest on deposits	$+rD$			$-rD$	0
Interest on loans		$-rL$		$+rL$	0
Profits		$-\Pi$	$+\Pi_u$		0
Sum	S_h	0	$S_f - pI$	0	0
Flow of Funds					
Deposits	$+\dot{D}$			$-\dot{D}$	0
Loans			$-\dot{L}$	$+\dot{L}$	0
Capital			$+pI$		pI
Sum	S_h	0	Π_u	0	pI
Change in Net Worth	S_h	$(S_f + \dot{p}K - p\delta K)$			$\dot{p}K + p\dot{K}$

Table: SFC table for the Keen model.

Keen model - Investment function

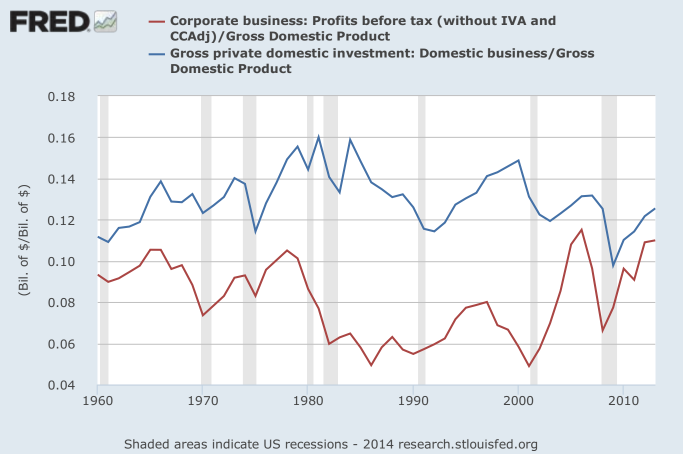
- Assume now that new investment is given by

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K$$

  where $\kappa(\cdot)$ is a nonlinear increasing function of profits $\pi = 1 - \omega - rd$.
- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y$$

Investment and profits, US 1960-2014



Keen model - Differential Equations

Denote the debt ratio in the economy by $d = D/Y$, the model can now be described by the following system

$$
\begin{aligned}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha\right] \\
\dot{\lambda} &= \lambda\left[\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \frac{\kappa(1-\omega-rd)}{\nu} + \delta\right] + \kappa(1-\omega-rd) - (1-\omega)
\end{aligned} \tag{1}
$$

Example 2: convergence to the good equilibrium in a Keen model

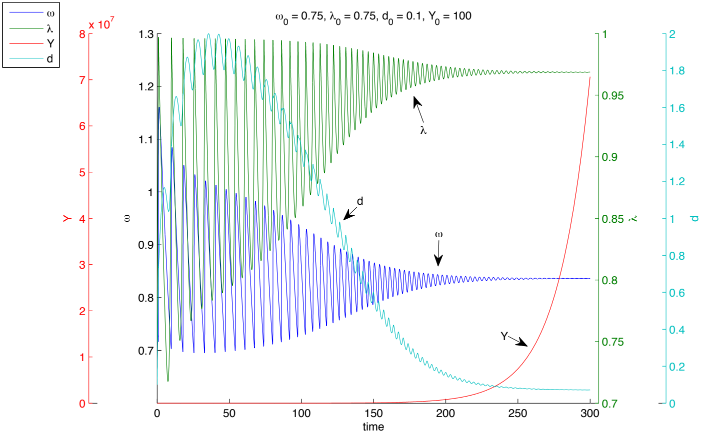


Figure: Grasselli and Costa Lima (2012)

Example 3: explosive debt in a Keen model

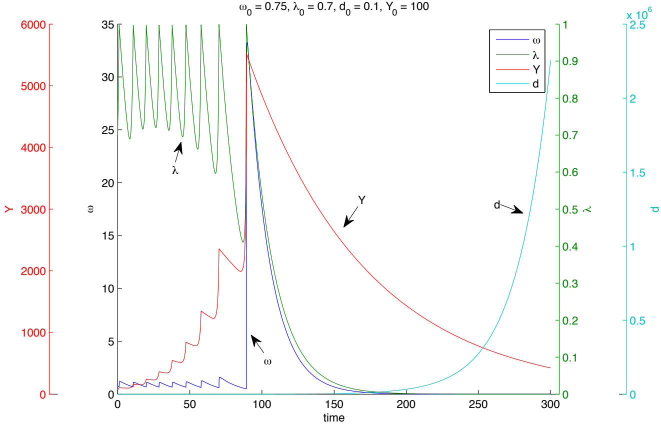
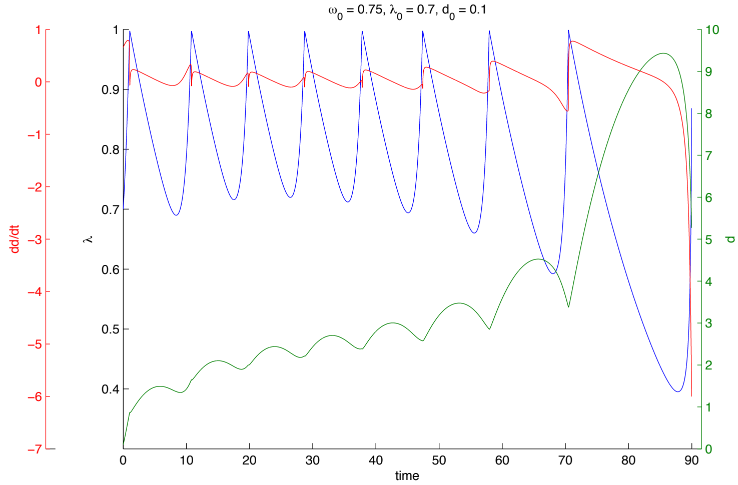
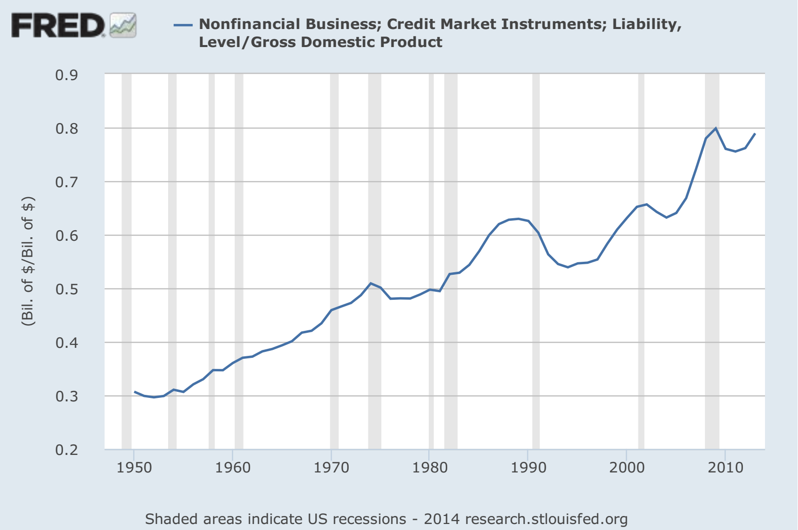


Figure: Grasselli and Costa Lima (2012)

Example 3 (continued): explosive debt in a Keen model



Corporate Debt share in the US 1950-2014



Private debt matters!

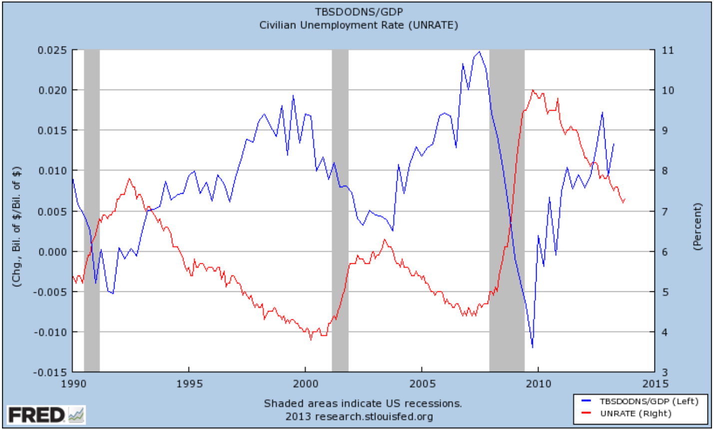


Figure: Change in debt and unemployment.

Basin of convergence for Keen model

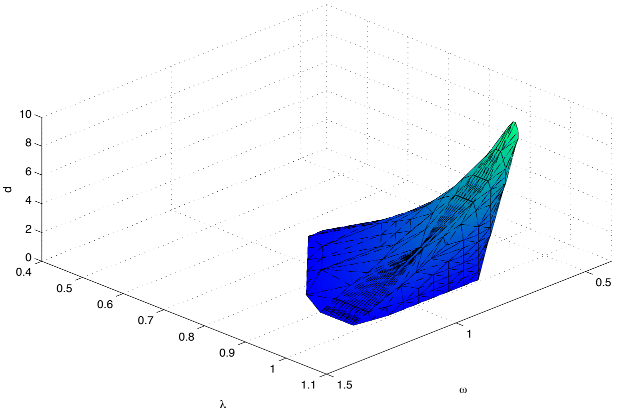


Figure: Grasselli and Costa Lima (2012)

Ponzi financing

To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y + P$$
$$\dot{P} = \Psi(g(\omega,d))P$$

where $\Psi(\cdot)$ is an increasing function of the growth rate of economic output

$$g = \frac{\kappa(1-\omega-rd)}{\nu} - \delta.$$

Example 4: effect of Ponzi financing

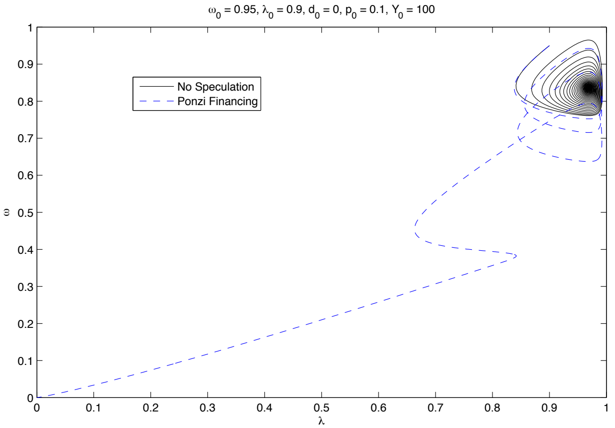


Figure: Grasselli and Costa Lima (2012)

Stock prices

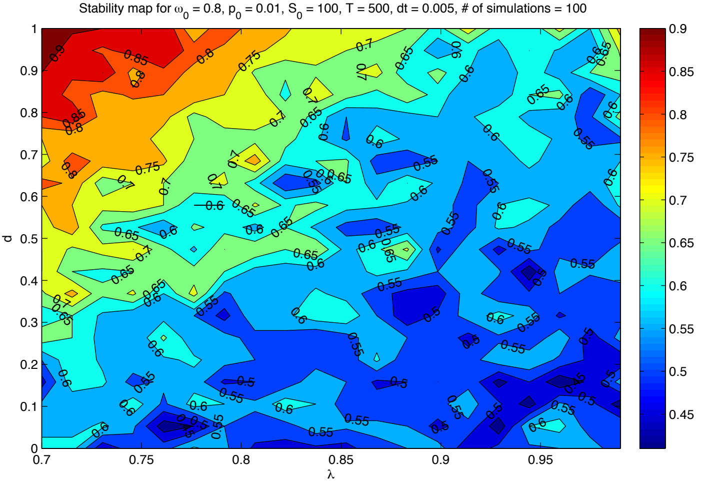
- Consider a stock price process of the form

$$\frac{dS_t}{S_t} = r_b dt + \sigma dW_t + \gamma \mu_t dt - \gamma dN^{(\mu_t)}$$

  where N_t is a Cox process with stochastic intensity $\mu_t = M(p(t))$.
- The interest rate for private debt is modelled as $r_t = r_b + r_p(t)$ where

$$r_p(t) = \rho_1 (S_t + \rho_2)^{\rho_3}$$

Stability map



The Great Moderation in the U.S. - 1984 to 2007

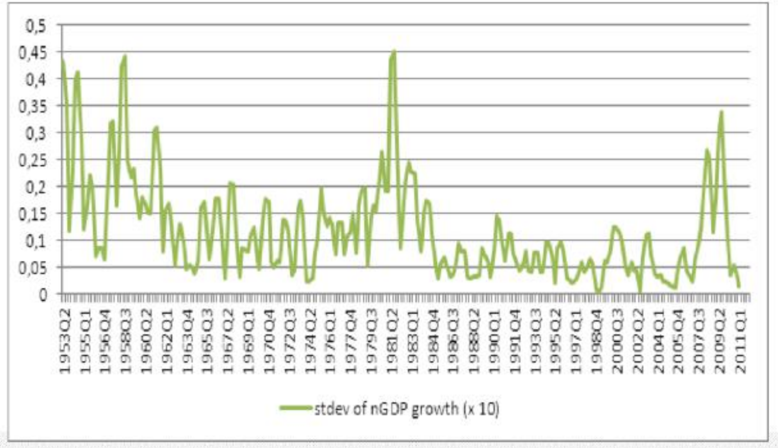


Figure: Grydaki and Bezemer (2013)

Possible explanations

- Real-sector causes: inventory management, labour market changes, responses to oil shocks, external balances , etc.
- Financial-sector causes: credit accelerator models, financial innovation, deregulation, better monetary policy, etc.
- Grydaki and Bezemer (2013): growth of debt in the real sector.

Bank credit-to-GDP ratio in the U.S

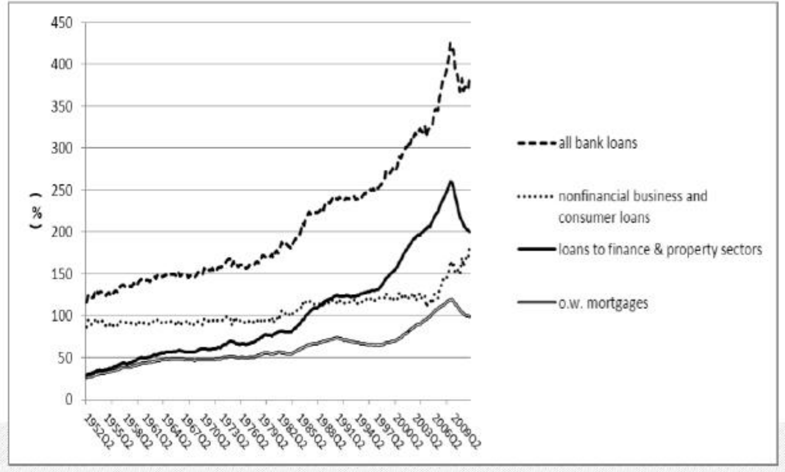


Figure: Grydaki and Bezemer (2013)

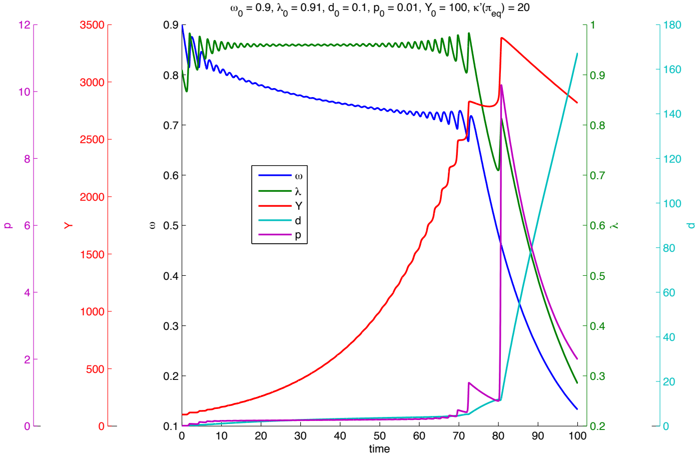
Excess credit growth moderated output volatility during, but not before the Great Moderation

Before the Great Moderation	*During the Great Moderation*
change in interest rate (-) => output volatility	excess credit growth (-) => output volatility
change in interest rate (+) => inflation	output volatility (+) => excess credit growth
excess credit growth (+) => change in interest rate	output volatility (-) => change in interest rate
	excess credit growth (+) => change in interest rate
	inflation (+) => change in interest rate

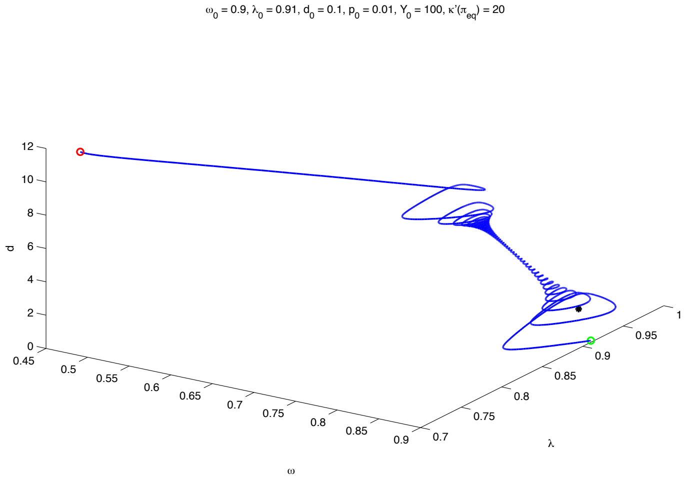
Note: In the table, x (-) => y denotes that a one-standard deviation shock in variable x impacts negatively on the change of variable y. Similarly, x (+) => y indicates a positive impact.

Figure: Grydaki and Bezemer (2013)

Example 5: strongly moderated oscillations



Example 5 (cont): Shilnikov bifurcation



Shortcomings of Goodwin and Keen models

- No independent specification of consumption (and therefore savings) for households:

$$C = W, \quad S_h = 0 \qquad \text{(Goodwin)}$$

$$C = (1 - \kappa(\pi))Y, \quad S_h = \dot{D} = \Pi_u - I \qquad \text{(Keen)}$$

- Full capacity utilization.
- Everything that is produced is sold.
- No active market for equities.
- Skott (1989) uses prices as an accommodating variable in the short run.
- Chiarella, Flaschel and Franke (2005) propose a dynamics for inventory and expected sales.
- Grasselli and Nguyen Huu (2014) provide a synthesis, including equities and Tobin's portfolio choices.

Price dynamics

- A general price-wage dynamics taking into account both labor costs and expected inflation takes the form

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \Phi(\lambda) + \eta_1 \frac{\dot{p}}{p} + \eta_2 i_e$$

$$\frac{\dot{p}}{p} = \Phi_p(c, p) + \eta_3 i_e$$

$$\frac{d}{dt}(i_e) = \eta_4 \left[\frac{\dot{p}}{p} - i_e\right],$$

- Here we assume the simplified version

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \Phi(\lambda) + \gamma \frac{\dot{p}}{p},$$

$$\frac{\dot{p}}{p} = -\eta_p \left[1 - m\frac{c}{p}\right]$$

for a constants $0 \leq \gamma \leq 1$, $\eta_p > 0$ and $m \geq 1$.

Inventory dynamics

- Denoting demand by $Y_d = C + I_k$, we postulate that expected sales evolve according to
$$\dot{Y}_e = (\alpha + \beta) Y_e + \eta_d (Y_d - Y_e).$$
- Moreover, we assume that the desired level of inventory is $V_d = f_d Y_e$ and that planned changes in inventory are given by
$$I_p = (\alpha + \beta) V_d + \eta_v (V_d - V).$$
- Finally, production is give by $Y = Y_e + I_p$, which in turn determines utilization through $u = Y/Y_{\max} = \nu Y/K$.
- To complete the specification of firm and household behaviour we set
$$I_k = \left[\frac{\kappa(\pi_e) + \eta_u (u - \overline{u})}{\nu}\right] K$$
$$pC = c_1 W + c_2 D$$

Extended System

Defining $\omega_p = W/(pY)$ and $d_p = D/(pY)$ leads to

$$
\begin{aligned}
\dot{\omega}_p =& \omega_p \left[\Phi(\lambda) - \alpha + (1-\gamma)\eta_p(1 - m\omega_p)\right] \\
\dot{\lambda} =& \lambda \left[g_e y_e + g_d y_d - \eta_v - \alpha - \beta\right] \\
\dot{d}_p =& d_p \left[r - g_e y_e - g_d y_d + \eta_v + \eta_p(1 - m\omega_p) - c_2\right] \\
& + (y_d - c_1)\omega_p \\
\dot{y}_e =& y_e(\alpha + \beta - \eta_d - g_e y_e - g_d y_d + \eta_v) + \eta_d y_d \\
\dot{u} =& u\left[g_e y_e + g_d y_d - \eta_v - y_d + c_1\omega_p + c_2 d_p + \delta\right]
\end{aligned},
$$

for constants g_e, g_d and with

$$
y_d = c_1\omega_p + c_2 d_p + \frac{\kappa(\pi_e) + \eta_u(u - \overline{u})}{u}.
$$

Firm decisions

- Suppose now that firms finance new investment by issuing equities E at price p_e as well as new loans.
- Assuming that undistributed profits take the form $s_f\Pi$ for a constant s_f, the amount needed to be raised externally for new investment is $pI_k - s_f\Pi$, according to the proportions

$$\dot{D} = \nu_D[pI_k - s_f\Pi]$$
$$p_e\dot{E} = \nu_E[pI_k - s_f\Pi],$$

with $\nu_D + \nu_E = 1$.

- Here both I_k and ν_E can be functions of Tobin's $q = \frac{p_eE}{pK}$.

Household decisions

- On the other hand, the budget constraint for households is

$$W + (1 - s_f)\Pi + rD = pC + \dot{D} + p_e\dot{E},$$

whereas their portfolio allocation is

$$\begin{aligned} p_e E &= f_e(r_e^e)X_h \\ D &= 1 - f_e(r_e^e)X_h, \end{aligned}$$

where

$$\begin{aligned} r_e^e &= \frac{(1 - s_f)\Pi}{p_e E} + \pi_e^e \\ \dot{\pi}_e^e &= \beta_{\pi_e}\left(\frac{\dot{p}_e}{p_e} - \pi_e^e\right) \end{aligned}$$

- This leads to an extended system with two more equations for $\dot{e}/e$ and $\dot{\pi}_e^e$.

Concluding remarks

- Macroeconomics is too important to be left to macroeconomists.
- Since Keynes's death it has developed in two radically different approaches:
    1. The dominant one has the appearance of mathematical rigour (the SMD theorems notwithstanding), but is based on implausible assumptions, has poor fit to data in general, and is disastrously wrong during crises. Finance plays a negligible role
    2. The heterodox approach is grounded in history and institutional understanding, takes empirical work much more seriously, but is generally averse to mathematics. Finance plays a major role.
- It's clear which approach should be embraced by mathematical finance.

Thank you!