February 24, 2014
KAP 414
2:00 PM

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"Bismut-Elworthy-Li Formulas for Diffusions with Irregular Drift Coefficients"

Abstract: One of the most pertinent applications of Malliavin calculus in Mathematical Finance is the representation of the so-called Greeks, which are sensitivities of option prices w.r.t. to involved parameters, as expectation functionals of the pay-off function times a so-called Malliavin weight. In particular for non-smooth pay-off functions this representation yields a numerically efficient way to compute Greeks. In the case of the Delta (sensitivity w.r.t. the initial value of the underlying diffusion), which is of special interest for hedging purposes, the above mentioned representation is also referred to as Bismut-Elworthy-Li type formula. However, in the existing literature the underlying Ito diffusion is assumed to have smooth coefficients. For example, an extended Ornstein-Uhlenbeck process with regime switching mean reversion rate, an important model in electricity price modelling, is not included in this class of Ito diffusions.

In this presentation we demonstrate how to generalize the Bismut-Elworthy-Li type formula to Ito diffusion with irregular drift coefficients. To this end, we study the theoretical questions of existence and Malliavin differentiability of strong solutions of stochastic differential equations with irregular drift coefficients. Using techniques from white noise analysis and a compactness criteria based on Malliavin calculus we develop a new method for the construction of strong solutions of SDEs with irregular drift coefficients. Further, this approach yields the additional important result that the constructed strong solutions are Malliavin differentiable and Sobolev differentiable in their initial conditions. This insight together with some local time variational calculus finally enables us to extend the corresponding Bismut-Elworthy-Li representation.