March 15, 2013
KAP 414
3:30 PM- 4:30 PM

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"Stable Diffusions with Rank-Based Interactions and Models of Large Equity Markets"

Abstract

We introduce and study ergodic diffusion processes interacting through their ranks. These interactions give rise to invariant measures which are in broad agreement with stability properties observed in large equity markets over long time-periods. The models we develop assign growth rates and variances that depend on both the name (identity) and the rank (according to capitalization) of each individual asset. Such models are able realistically to capture critical features of the observed stability of capital distribution over the past century, all the while being simple enough to allow for rather detailed analytical study.

The methodologies used in this study touch upon the question of triple points for systems of interacting diffusions; in particular, some choices of parameters may permit triple (or higher-order) collisions to occur. We show, however, that such multiple collisions have no effect on any of the stability properties of the resulting system. This is accomplished through a detailed analysis of collision local times. The models have connections with the analysis of Queuing Networks in heavy traffic, and with competing particle systems in Statistical Mechanics (e.g., Sherrington- Kirkpatrick model for spin-glasses). Their hydrodynamic-limit behavior is governed by generalized porous medium equations with convection, whereas limits of a different kind display phase transitions and are governed by Poisson-Dirichlet laws. We survey briefly recent progress on some of these fronts.