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"Near-Expiry Asymptotics of the Implied Volatility in Local and Stochastic Volatility Models"

Abstract: Implied volatility in diffusion type stock and volatility models as a function of time and initial parameters (initial stock and volatility) usually leads to a quasilinear parabolic differential equation. Such an equation does not seem to be well suited for studying asymptotic behavior of the implied volatility. We propose a probabilistic method based the asymptotic expansion of the transition density function of the stock-volatility process. In the case of the SABR stochastic volatility model, the underlying geometric structure is hyperbolic and, because of the absence of cutlocus, the asymptotic expansion of the transition density behaves especially well. Theoretically speaking, this method leads to a full asymptotic expansion of the implied volatility near the expiration time.   In practice it is particularly effective strictly away from the strike price and in this case one can calculate explicitly the first two or three terms of the asymptotic expansion, depending on the complexity of the model under consideration. With modifications the method can also be applied to other parameter regimes such as when the initial stock price is near the strike price.