March 11, 2013
KAP 414
3:30 PM- 4:30 PM

Integral Representation of Martingales and Endogenous Completeness in Financial Economics

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Abstract

Let $\mathbb{Q}$ and $\mathbb{P}$ be equivalent probability measures and let ψ be a J-dimensional vector of random variables such that $\frac{d\mathbb{Q}}{d\mathbb{P}}$ and ψ are defined in terms of a weak solution X to a d-dimensional stochastic differential equation. Motivated by the problem of *endogenous completeness* in financial economics we present conditions which guarantee that every local martingale under $\mathbb{Q}$ is a stochastic integral with respect to the J-dimensional martingale $S_t := \mathbb{E}^{\mathbb{Q}}[\psi|\mathcal{F}_t]$. While the drift $b = b(t,x)$ and the volatility $\sigma = \sigma(t,x)$ coefficients for X need to have only minimal regularity properties with respect to x, they are assumed to be analytic functions with respect to t. We provide a counterexample showing that this t-analyticity assumption for σ cannot be removed.

The presentation is based on a joint work with Sivliu Predoiu; see http://arxiv.org/abs/1110.3248.