Prof. Carl Mueller
University of Rochester

DO STOCHASTIC PDE HIT POINTS OR HAVE DOUBLE POINTS IN THE CRITICAL DIMENSIONS?

This talk will describe work in progress with R. Dalang, Y. Xiao, and S. Tindel. The stochastic heat equation is often used as a basic model for a moving polymer:

$$(1) \qquad \partial_t u = \Delta u + \dot{W}(t,x).$$

Here, $u = u(t,x) \in \mathbf{R}^d$ is the position of the polymer, $x \in \mathbf{R}$ is the length along the polymer, and t is time. $\dot{W}(t,x)$ is two-parameter vector-valued white noise. Note that $u \in \mathbf{R}^d$; that is, u is vector-valued. This is consistent with the interpretation of u as the position of a polymer.

We say that the solution hits a point z if there is positive probability that $u(t,x) = z$ for some (random) parameters (t,x). We say that the solution has a double point if there exist distinct pairs of parameters (t,x) and (s,y) with $t,s > 0$, $x,y \in \mathbf{R}$ such that $u(t,x) = u(s,y)$. Some time ago the speaker and R. Tribe proved that $d = 6$ is the critical dimension for the solution u to hit points. That is, for a generic point z, the solution u hits points iff $d < 6$. In the same way, the critical dimension for double points is $d = 12$. The proofs use arguments which are specific to (1).

The goal of our work, which is still in progress, is to adapt an argument of Talagrand to study this question for equations similar to (1) but which

(1) have colored noise in place of white noise.
(2) have nonlinearities.

As usual, the critical case is by far the hardest. In fact, there are a number of results about the situation away from criticality, but they are not sharp enough to give the results we seek.