Insiders' Hedging in a Stochastic Volatility Model with Informed Traders of Multiple Levels

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Outline

Market Microstructure

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· micro vs. macro $\leftarrow$ surface of the land vs. earth from the space

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· HFD, UHFD, algorithmic trading
· transaction cost, fees, taxes, regulations
· financial engineering vs. economics ← Who determines the price?
· information, liquidity(or liquidation)
· CAPM, Nash equilibrium etc.

Information Asymmetry

Background

· In a market, different traders have different levels of information.
· Even when two traders have the exactly same information, they may interpret the information in different ways, or make different decisions.
· Information is modeled by a filtration in mathematical finance theory.
· A trader with more information has a larger filtration than a trader with less information.

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Traders

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How to model?

· We introduce an information process.
· This exclusive information often causes bigger movements than those usual diffusion can explain, and it is natural to involve this information to jump terms.
· jump in the price process itself? jump in the volatility term? jump size? jump timing(intensity)?

Filtration

· Let $\mathcal{F}$ be the filtration generated by the market. It is an honest trader's filtration.
· An insider has a larger filtration $\mathcal{G}$ available only to insiders.
· $\mathcal{F} \subset \mathcal{G}$.
· Kyle(1985), Amendinger(2000), Biagini and Oksendal(2005) assumed that the $\mathcal{G}_t = \mathcal{F}_t \vee \sigma(L)$ for some fixed random variable L. (usually a future price)
· Hu and Oksendal(.) studied a model that more and more additional information is available to the investor as time goes by. They used a sequence of random variables available only to insiders as additional information at certain points of times.(scheduled announcements)
· We generalize these studies to the case with $\mathcal{G}_t = \mathcal{F}_t \vee \sigma(X_s, 0 \leq s \leq t)$, where the additional information X given to insiders is not a single random variable nor a discrete sequence of random variables, but a diffusion process.

Research on Information Effects

Q: Obviously, an informed trader should do better in the market. But how can we mathematically explain and support this? More specifically, how can we find an optimal hedging strategy and pricing for an informed trader?

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Earning announcement and earning jumps

· Lee and Leung
· deterministic time jump
· learning procedure $\rightarrow$ Brownian bridge

Research on Information Effects

Q: Other information issues

more finance* papers

· K.Lee, R.Christie-David, A. Chatrath and B.Adrangi(Journal of Futures Markets, Volume 31, Issue 10, pages 915-946, October 2011)
→ Dominant markets, staggered openings, and price discovery
→ Spillover effect, leading-following interaction
· K.Lee, R.Chrisite-David and A. Chatrath(Journal of Futures Markets, vol 29, (1), 42-73, 2009):
→ How potent are news reversals?: Evident from futures markets
→ Surprise!

Lee and Song(2007)

$$dS_t = f(S_{t-})dB_t + g(S_{t-})dR_t + h(S_{t-})dt \tag{1}$$
$$0 \le t \le T$$

where

- $R_t = \sum_{n=1}^{N_t} U_n$
- $N_t - \int_0^t \lambda(X_s)ds =$ a local martingale under $\mathbf{P}$
- X, which is a firm specific information available only to insiders, satisfies the stochastic differential equation $dX_t = \alpha(X_t)dt + \beta(X_t)dB_t^X$ for $0 \le t \le T$.
- B' is another standard Brownian motion under $\mathbf{P}$ such that $[B, B^X]_t = \rho t$.
- Correlation ρ between two Brownian motions B and B^X explains the level of exclusive information.
- U_n is i.i.d and has a pdf ν on $(-1, 1)$
- U_n denotes the jump sizes of S_t and has mean 0 and a finite second moment σ^2.

Kang and Lee(2014)

$$dS_t = S_{t-}(\mu dt + \sigma dB_t + dR_t), 0 \leq t \leq T \tag{2}$$

where

- B_t is a standard Brownian motion.
-
$$R_t = \sum_{0<s\leq t} \theta(X_s)1(\Delta N_s = 1)$$

where $\theta(\cdot)$ is an increasing function and $-1 < \theta(x) < \frac{\sigma^2}{\mu}$.

- N_t is a Poisson counting process with rate λ under $\mathbf{P}$. $\hat{N}_t := N_t - \lambda t$ is a martingale under $\mathbf{P}$.
-
$$dX_t = \alpha(X_t)dt + \beta(X_t)dB_t^X, \quad X_0 = x_0.$$

where B^X is a standard Brownian motion with $[B, B^X]_t = \rho t$.

Distribution of Jump Sizes for $\alpha(x) = 0$, $\beta(x) = 1$.

The expectation of jump size is given by $E[\theta(X_0 + \sqrt{T}Z)]$ where T and Z follow independent exponential with rate λ and standard normal distribution respectively. $\theta(x) = \frac{2}{\pi}\arctan(x)$

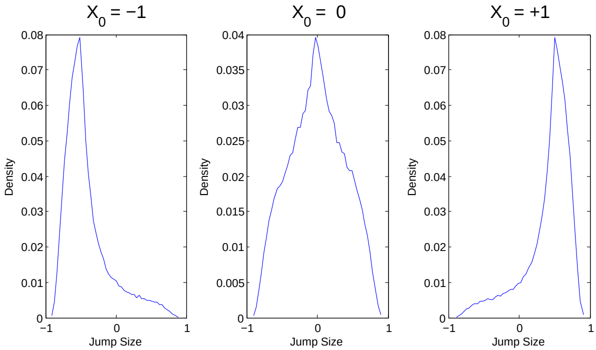

Figure: Jump distributions for different X_0's.

Comparison with Honest Trader's Strategy

We assume that an honest trader believes the Black-Scholes model. The first number in a cell denotes the expected total cost of the informed trader, and the second number denotes that of the honest trader. $E[(C_T - C_0)^2]$ denotes the expected total cost, which will be explained in 3 slides. (A smaller number is better!)

Table: $E[(C_T - C_0)^2]$, $\rho = -0.5$

Vol	10%	20%	30%
Vol Ratio	1.934157	1.299467	1.154251
80	0.104438, 0.751323	1.028191, 1.189337	1.851046, 1.792153
100	2.036793, 4.945836	1.537828, 1.686250	4.415904, 4.045074
120	0.721526, 0.922125	1.702788, 2.112400	2.645012, 1.369176

Table: $E[(C_T - C_0)^2]$, $\rho = 0.0$

Vol	10%	20%	30%
Vol Ratio	1.988265	1.317293	1.148387
80	0.080125, 1.069857	0.269744, 0.830191	0.962809, 1.119722
100	1.568441, 5.419202	1.047886, 1.557627	1.606752, 1.889270
120	0.693646, 2.347789	1.366413, 2.494248	1.683573, 1.805261

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Vol	10%	20%	30%
Vol Ratio	1.981465	1.318082	1.157330
80	0.611927, 1.590476	0.289834, 0.699058	0.814961, 1.101575
100	0.397466, 2.538860	1.052195, 1.639205	1.729693, 1.940148
120	1.072975, 1.680556	1.362013, 1.727360	1.875722, 1.793749

Park and Lee(2016?)

- $$dS_t = \mu_0 S_{t-} dt + \sigma S_{t-} dB_t + S_{t-} dR_t, \quad S_0 = s$$
- $$dX_t = \alpha(X_t)dt + \beta(X_t)dB_t^X, \quad X_0 = 1$$

where W^X is a standard Brownian motion.

- Define
$$R_t = \int_0^t \int_{-\infty}^{\infty} y p^R(X_s, dy, ds),$$
where $p^R(X_t, dy, dt)$ is a random measure on $R \times [0, T]$.

- Also, we assume that there exists a compensated measure $m_1(X_t, dt)$ such that
$$E[\int_0^T C_s dR_t] = E[\int_0^T \int_R C_s(y) m_1(X_s, dy, ds)]$$
for all nonnegative $\mathcal{F}_t$-adapted processes C_t.

Multi-Level Traders

Idea

· multiple information processes → vector process
· several levels within informed traders
· hard to model a price process with multiple jumps → volatility factors in stochastic volatility model

Basics

· We consider a market with one risky asset (S_t) and one riskless asset which would be assumed 1.
· Portfolio: a pair of processes (ξ_t, η_t), $V_t = \xi_t S_t + \eta_t$
· Contingent claim: $H = H(S_T)$ at time T.
· Cost process of a portfolio (ξ_t, η_t): $C_t = V_t - \int_0^t \xi_u dS_u$, $0 \le t \le T$

Hedging(replicating)

A (prefect) hedging portfolio(strategy) for a contingent claim $H(S_T)$ should satisfy the following two conditions.

1. Self-financing:

$$V_t = \xi_t S_t + \eta_t = \xi_0 S_0 + \eta_0 + \int_0^t \xi_u dS_u$$

2. Perfect match at maturity: $H(S_T) = V_T$

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• For a self financing portfolio, the cost process $C_t = V_t - \int_0^t \xi_u dS_u = \xi_0 S_0 + \eta_0 = C_0$ is a constant for all t.

• A complete market is a market where every contingent claim has a hedging portfolio. (ex. Black-Scholes model)

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• A complete market is a market where every contingent claim has a hedging portfolio. (ex. Black-Scholes model)
• On the other hand, in an incomplete market, there is no strategy which satisfies both conditions.
Q: Then what is a 'good' hedging strategy in an incomplete market?

Model

Model

$$dS_t = \mu S_t dt + f(\mathbf{Y_t}) S_t dW_t^{(0)}, \tag{3}$$

$$dY_t^{(i)} = \alpha_i(t, Y_t^{(i)})dt + \beta_i(t, Y_t^{(i)})dW_t^{(i)} + \gamma_i(t, Y_t^{(i)})dR_t^{(i)}, \ i = 1, \cdots, n. \tag{4}$$

on a $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \le t \le T}, \mathbf{P})$ where $\mathbf{P}$ is the empirical probability measure, and $\mathbf{Y} = (Y^{(1)}, \cdots, Y^{(n)})$.

· $R_t^{(i)} = \sum_{j=1}^{N_t^{(i)}} U_j^{(i)}$.
· $U_j^{(i)}$: i.i.d. random variables with densities ν_i,
· $E[U_j^{(i)}] = 0$ and $E[|U_j^{(i)}|^2] = \eta_i^2$.
· $N^{(i)}$: a Poisson process with bounded intensity λ_i.
· ρ_{ij} : correlation between $W^{(i)}$ and $W^{(j)}$

- different types of information: scheduled, randomly arriving, continuous etc.

Basic Assumptions

Notations

· $\mathbf{S}_t$ is the solution vector $(S_t, Y_t^{(1)}, \cdots, Y_t^{(n)})$.
· $\mathbf{M}_t$ denotes the martingale part of $\mathbf{S}_t$ i.e.

$$\mathbf{M}_t = (\int_0^t fS_s dW_s^{(0)}, \int_0^t (\beta_1 dW_s^{(1)} + \gamma_1 dR_s^{(1)}), \cdots, \int_0^t (\beta_n dW_s^{(n)} + \gamma_n dR_s^{(n)}))$$

Basic assumptions

· spot rate of interest $r = 0$ and no dividend.
· The volatility function f is always positive.
· $\mathbf{S}_t$ is a $\mathcal{H}^2$ special semimartingale with the canonical decomposition $\mathbf{S}_t = \mathbf{M}_t + \mathbf{A}_t$ and $\mathbf{M}_t$ is a square-integrable martingale under $\mathbf{P}$. In other words,

$$\|[\mathbf{M}, \mathbf{M}]_T^{1/2}\|_{L^2}^2 < \infty \tag{5}$$

$$\| \int_0^T |\alpha_i(t, Y_t^{(i)})| dt \|_{L^2}^2 < \infty, \quad i = 1, \cdots, n. \tag{6}$$

Minimal Martingale Measure

- pricing point of view, the second fundamental theorem
- useful to find the Föllmer-Schweizer decomposition

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· useful to find the Föllmer-Schweizer decomposition

Definition

A martingale measure $\mathbf{Q}$ which is equivalent to $\mathbf{P}$ is called *minimal* if $\mathbf{Q} = \mathbf{P}$ on $\mathcal{F}_0$, and if any square-integrable $\mathbf{P}$-martingale L that satisfies $\langle L, M \rangle = 0$ remains a martingale under $\mathbf{Q}$, where M is the martingale part of S in the canonical decomposition under $\mathbf{P}$.

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Theorem

Let

$$X_t = \int_0^t \frac{\mu}{f(\mathbf{Y_s})} dW_s^{(0)}, \tag{7}$$

and assume that $E[e^{2X_t}] < \infty$ *for evert* $t \leq T$. *Then,*

$$Z_t = 1 - \int_0^t Z_{s-} dX_s \tag{8}$$

is a $\mathbf{P}$*-martingale and the probability measure* $\mathbf{Q}$ *defined by* $d\mathbf{Q} = Z_T d\mathbf{P}$ *is the minimal martingale measure of* S.

Minimal Martingale Measure

Idea of the Proof:

- Doob Meyer Decomposition of M_t
- Girsanov-Meyer theorem
- Kunita-Watanabe inequality
- Uniqueness of SDE
- Stochastic Exponential
- condition on a local martingale to be a true martingale

Q Dynamics of BMs

Lemma

Under the minimal martingale measure $\mathbf{Q}$,

$$\widetilde{W}_t^{(0)} := W_t^{(0)} + \int_0^t \frac{\mu}{f(\mathbf{Y_s})} ds,$$

$$\widetilde{W}_t^{(i)} := W_t^{(i)} + \rho_{oi} \int_0^t \frac{\mu}{f(\mathbf{Y_s})} ds \quad i = 1, 2, \cdots, n$$

are Brownian motions under $\mathbf{Q}$*. Thus* $\mathbf{S}$ *satisfies SDEs*

$$\begin{aligned} dS_t &= f(\mathbf{Y_t}) S_t d\widetilde{W}_t^{(0)} \\ dY_t^{(i)} &= (\alpha_i(t, Y_t^{(i)}) - \mu\rho_{oi} \frac{\beta_i(t, Y_t^{(i)})}{f(\mathbf{Y_s})}) dt + \beta_i(t, Y_t^{(i)}) d\widetilde{W}_t^{(i)} + \gamma_i(t, Y_{t-}^{(i)}) dR_t^{(i)} \end{aligned} \tag{9}$$

under measure $\mathbf{Q}$.

$\mathbf{Q}$ Dynamics of $R^{(i)}$

$\mathbf{Q}$ dynamics of $R^{(i)}$

: Let $p_{R^{(i)}}(dt, dy_i)$ be the random measure associated to the jump process $R^{(i)}$ under $\mathbf{P}$. Then, the compensated measure of $R^{(i)}$ under $\mathbf{Q}$ is given by

$$p_{\widetilde{R}^{(i)}} = p_{R^{(i)}}(dt, dy_i) - \lambda_i \nu_i(dy_i)dt \tag{10}$$

→ characteristics of semimartingale
→ Girsanov's theorem for random measures
→ conditional expectation with respect to predictable σ-field

Local Risk Minimization Strategy

Value process

· ξ_t : the amount of the underlying asset
· η_t : the amount of the money market account
· V_t : the value process of a portfolio (ξ, η) defined by $V_t = \xi_t S_t + \eta_t$

Cost process

· C_t : the cost process defined by $C_t = V_t - \int_0^t \xi_t dS_t$

Local risk minimization strategy in an incomplete market (Föllmer and Schweizer)

· local risk minimization strategies ξ_t : The cost process C is a square integrable martingale orthogonal to M, i.e. $\langle C, M \rangle_t = 0$ where M is the martingale part of S under $\mathbf{P}$.

Local Risk Minimization Strategy

A sufficient condition for the existence

The existence of an optimal strategy is equivalent to a decomposition

$$H = V_0 + \int_0^T \xi_u^H \, dS_u + L_T^H$$

where L_t^H is a square integrable martingale orthogonal to M_t. For such a decomposition, the associated optimal strategy (ξ_t, η_t) is given by $\xi_t = \xi_t^H$, $\eta_t = V_t - \xi_t S_t$, where $V_t = V_0 + \int_0^t \xi_u^H \, dS_u + L_t^H$.

Local Risk Minimization Strategy

Computation of the optimal strategy

Suppose that $V_t = E^{\mathbf{Q}}[H(S_T)|\mathcal{G}_t]$ has a decomposition

$$V_t = V_0 + \int_0^t \xi_u^H dS_u + L_t$$

where L_t is a square integrable $\mathbf{P}$ martingale such that $\langle L, M\rangle_t = 0$ under $\mathbf{P}$. Then ξ_t^H is given by

$$\xi^H = \frac{d\langle V, S\rangle}{d\langle S, S\rangle}. \tag{11}$$

where the conditional quadratic variations are calculated under $\mathbf{P}$.

· role of the minimal martingale measure(L_t)

Different traders

Different Traders

· A level k trader : a trader with information $Y^{(1)}, Y^{(2)}, ..., Y^{(k)}$, $k = 1, 2, \cdots, n$
· A level n trader : a fully informed trader
· A level 0 trader : honest trader, uninformed trader, noise trader, liquidity trader

Filtration

· $\mathcal{G}_t^{(k)} = \sigma\{(S_s, Y_s^{(1)}, \cdots, Y_s^{(k)}), 0 \leq s \leq t\}$
· $\mathcal{G}_t^{(0)} \subset \mathcal{G}_t^{(1)} \subset \cdots \subset \mathcal{G}_t^{(n)} \subset \mathcal{F}_t$

Local Risk Minimization Strategy for a Fully Informed Trader

Consider a European syle contingent claim $H(S_T) \in \mathbb{L}^2(\mathbf{P})$

The fully informed trader

Let $V_t^{(n)} = E^{\mathbf{Q}}[H(S_T)|\mathcal{G}_t^{(n)}]$ be a price process of a fully informed trader .

Theorem

The local risk minimization strategy is given by

$$\xi_t^{n,H} = \frac{\partial V^{(n)}}{\partial S_t} + \frac{\sum_{i=1}^n \rho_{0i}\beta_i(t, Y^{(i)})\frac{\partial V^{(n)}}{\partial y_i}}{f(\mathbf{Y}_t)S_t}. \tag{12}$$

Local Risk Minimization Strategy for a Fully Informed Trader

Idea of the proof

· Expand $V_t^{(n)} = E^{\mathbf{Q}}[H(S_T)|\mathcal{G}_t^{(n)}]$ using the Markov property and Ito's formula.
· How to change the jumps in terms of integrals? $\rightarrow$ no common jumps!
· $V_t^{(n)}$ is a $\mathbf{Q}$ martingale, so the drift term of the expansion should be 0. This gives us the pricing differential equation as well as the representation of $V_t^{(n)}$.
· Calculate the Radon-Nikodym derivative to get $\xi_t^{n,H}$, using properties of the predictable version of quadratic variation.

Underlying Dynamic of a Level k Trader

Let $\sigma_0 := E^{\mathbf{Q}}[f(\mathbf{Y_t})] \geq 0$. Then price process becomes

$$\frac{1}{S_t} dS_t = \sigma_0 d\widetilde{W}_t^0 + \widetilde{f}(\mathbf{Y_t}) d\widetilde{W}_t^0 \tag{13}$$

where $\widetilde{f} := f - \sigma_0$.

Underlying of a level k trader

They can't observe all the information. So, $\widetilde{f}(\mathbf{Y_t})$ is not their volatility function. Define

$$\widetilde{f}_k(Y_t^{(1)}, \cdots, Y_t^{(k)}) := \widetilde{f}(Y_t^{(1)}, \cdots, Y_t^{(k)}, \tilde{y}_{k+1}, \cdots, \tilde{y}_n) \quad k = 1, \cdots, n$$

and $\widetilde{f}_k := 0$ if $k = 0$. Here, $(\tilde{y}_{k+1}, \cdots, \tilde{y}_n)$ is a constant vector. So a level k trader's price process (**??**) becomes

$$\frac{1}{S_t} dS_t = \sigma_0 d\widetilde{W}_t^0 + \widetilde{f}_k(\mathbf{Y_t}) d\widetilde{W}_t^0 \tag{14}$$

Cost Process of a Level k Trader

Cost process of a level k trader

· $V^{(k)}(t, S_t)$: the value process of a level k trader.
· $(\xi^{(k)}, \eta^{(k)})$: the portfolio of a level k trader
· $C^{(k)}$: the cost process of a level k trader defined by $C_t^{(k)} = V_t^{(k)} - \int_0^t \xi_t^{(k)} dS_s$

Local Risk Minimization Strategy for a Level k Trader

Theorem

The local risk minimization strategy for a level k trader is given by

$$\xi_t^{k,H} = \frac{\partial V^{(k)}}{\partial \widetilde{S}_t} + \frac{\sum_{i=1}^{k} \rho_{0i} \beta_i(t, Y^{(i)}) \frac{\partial V^{(k)}}{\partial y_i}}{f_k(\mathbf{Y}_t)\widetilde{S}_t}. \tag{15}$$

Note that the level 0 trader case corresponds to the B.S. hedging strategy $\frac{\partial V^{(k)}}{\partial \widetilde{S}_t}$.

The Optimal Choice for a Level k Trader

Assumption

· A level k trader wants to reduce the error in hedging.
· A level k trader has to choose a proper $f_k \rightarrow$ choose proper values for $(\tilde{y}_{k+1}, \cdots, \tilde{y}_n)$

Error function

· $\Theta := V^{(k)} - V^{(n)}$: an error function of a level k trader.

Theorem

Assume that $V^{(n)}(t, \mathbf{s})$ are in $C^{1,2}$. Then there exists a constant C which depends on a contingent claim $H(S_T)$ such that we have

$$E^{\mathbf{Q}}[|V_t^{(k)} - V_t^{(n)}|] \leq CE^{\mathbf{Q}}[\int_t^T |f_k(\mathbf{Y}_s) - f(\mathbf{Y}_s)|^2 ds]^{1/2} \tag{16}$$

The Optimal Choice for a Level k Trader

Optimal condition

$$E^{\mathbf{Q}}[f_k(\mathbf{Y}_s) - f(\mathbf{Y}_s)] = 0, \quad \text{for } t \le s \le T \tag{17}$$

Example (A special case)

We assume that $\mathbf{Y}_t$ is a $\mathbf{Q}$-martingale and f is a linear function $f = \sum_{i=1}^n c_i y_i$, where $c_i > 0$.

Under these conditions, $E^{\mathbf{Q}}[f(\mathbf{Y}_s)] = f(E^Q[\mathbf{Y}_s]) = f(\mathbf{Y}_0)$. Therefore, the choice

$$(\tilde{y}_{k+1}, \cdots, \tilde{y}_n) := (E^{\mathbf{Q}}[\mathbf{Y}_t^{(k+1)}], \cdots, E^{\mathbf{Q}}[\mathbf{Y}_t^{(n)}])$$

is the minimizer.

For example, $f(\mathbf{y}) = \sum_{i=1}^n y_i$ and $dY_t^{(i)} = \sqrt{Y_t^{(i)}} d\widetilde{W}_t^{(1)}$ satisfy all the conditions. Therefore, $\sigma_0 := \sum_{i=1}^n E^{\mathbf{Q}}[Y_t^{(i)}] = \sum_{i=1}^n Y_0^{(i)}$ and the optimal of f_k is $f_k := \sum_{i=1}^k y_i + \sum_{i=k+1}^n Y_0^{(i)}$.

An Example

We consider two information processes

$$dY_t^{(1)} = m_1 d\widetilde{W}_t^{(1)} + m_2 dR_t^{(1)} \tag{18}$$

$$dY_t^{(2)} = m_3 d\widetilde{W}_t^{(2)} + m_4 dR_t^{(2)}, \tag{19}$$

where each m_i is a given constant.

· $R_t^{(i)}$: uniformly distributed jumps with bounded intensities $\lambda_1 = 4$ and $\lambda_2 = 2$.
· $\sigma_0 = 0.2$, $m_1 = 0.1$, $m_2 = 0.05$, $m_3 = 0.05$, $m_4 = 0.1$, $\rho_{01} = \frac{1}{4}$, $\rho_{02} = \frac{1}{5}$ and $\rho_{12} = \frac{1}{20}$.
· $Y_0^{(1)} = Y_0^{(2)} = 0$ and volatility functions are $f(y_1, y_2) = \sigma_0 + y_1 + y_2$, $f_1(y_1, y_2) = \sigma_0 + y_1$.

Numerical Result for a Call Option

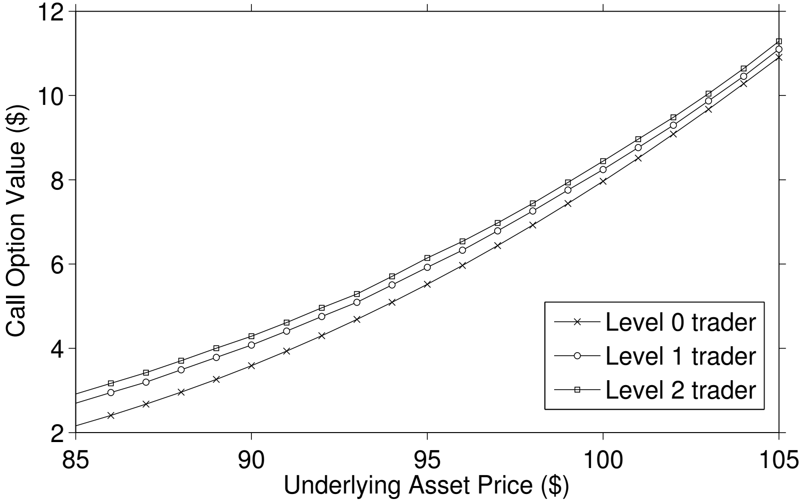

Figure: Call Price, $\sigma_0 = 0.2$, $K = 100$, $T = 1$

Numerical Result for a Call Option

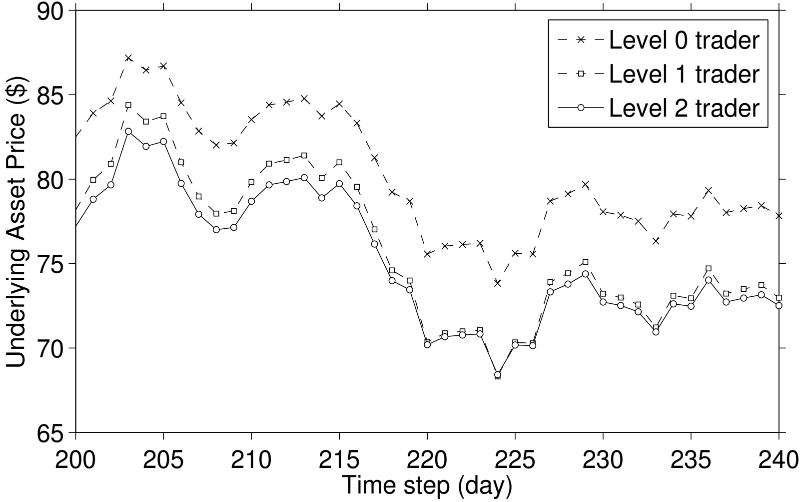

Figure: Sample Path of the Underlying

Numerical Result for a Call Option

$E[(C_T - C_0)^2]$		
Level 0 trader	Level 1 trader	Level 2 trader
7.728860384	3.533073956	2.714644221

Table: Expected total cost : $S_0 = 100$, $\sigma_0 = 0.2$, $K = 100$, $T = 1$, $dt = 1/100$

$E[(C_T - C_0)^2]$		
Level 0 trader	Level 1 trader	Level 2 trader
2.2653	2.2360	1.8912

Table: Expected total cost : $S_0 = 90$, $\sigma_0 = 0.2$, $K = 90$, $T = 1$, $dt = \frac{1}{50}$

$E[(C_T - C_0)^2]$		
Level 0 trader	Level 1 trader	Level 2 trader
0.6429	0.6136	0.5127

Table: Expected total cost : $S_0 = 90$, $\sigma_0 = 0.2$, $K = 90$, $T = 1$, $dt = \frac{1}{100}$

Summary

How the Information Works in a Trading?

· A trader with more information should do better in trading. We introduced those models in several cases. (jump size, timing, etc)
· We focused on a market with multiple levels of information processes.
· A numerical study shows mixed results. It is not clear how much advantage a trader gets by observing one more information process.

What to do next?

· more microstructure $\rightarrow$ algorithmic trading/ HFT
· other problems on information asymmetry
· uninformed or less informed trader's learning dynamic
· real data fitting??