Circuit Breakers and Contagion

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October 22, 2018
USC

Circuit Breakers

- Circuit breaker: When the decline in an index reaches a certain threshold, trading in a market is temporarily halted for a period of time.
- Commonly cited rationales: prevent stock market crashes and reduce excess volatility.
- Two direct effects:
  1. Market closure effect
  2. Price limit effect

Examples

- CB in the US market: when S&P 500 falls by 7% from the previous day close, market is halted by 15 minutes; when S&P 500 falls by 13%, market is halted for another 15 minutes; when S&P 500 falls by 20%, market is halted for the rest of the day.

- CB in the Chinese market: when CSI falls by 5% (Level I), market is halted by 15 minutes; when CSI falls by 7% (Level II), market is halted for the rest of the day.

The Case of Chinese Circuit Breakers

January 4, 2016 was the first day of introducing circuit breakers to the Chinese stock markets.

- January 4: At 13:13, Level I was triggered. At 13:28, the market opened again and Level II was triggered after only 6 minutes.
- January 5-6: The markets recovered weakly.
- January 7: At 09:42, Level I was triggered. At 09:57, the market opened again and level II was triggered 3 minutes later.
- Circuit breakers were abandoned since January 8, 2016.

Evidence of Contagion on January 4, 2016

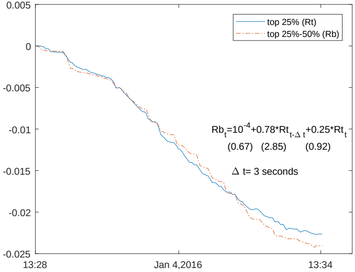

Figure 1: Evidence of contagion On January 4, 2016 in Chinese markets.

Literature Review

- Greenwald and Stein (1991) , Subrahmanyam (1994), Bernardo and Welch (2004)
- Chen, Petukhov and Wang (2018) : circuit breakers can increase volatility and increase the probability of reaching the threshold. In their model,
    1. there is a single stock in the market
    2. leverage unrestricted before market closure but prohibited after
    3. circuit breaker does not have price limit effect

Focus of this paper: Cross-stock impact

We develop a dynamic equilibrium model to study the cross-stock impact of circuit breakers in a market with multiple stocks.

Main questions include:

- What are effects of circuit breakers on the correlations among stocks?
- Can circuit breakers cause crash contagion?
- Can circuit breakers cause volatility contagion?
- Can circuit breakers accelerate market-wide decline?

Main Results

We find that circuit breakers may

- significantly increase correlations among stocks
- cause both crash contagion and volatility contagion
- accelerate price falling as the index approaches the circuit breaker threshold
- increase the probability of going down to the threshold

The Model

- Two agents A and B can continuously trade two stocks and one risk-free asset (with interest rate normalized to zero) from time 0 to T.
- The total supply of each stock is 1 and the total supply of the risk-free asset is zero. For $j \in \{A, B\}$, Agent j is initially endowed with no risk-free asset and $\theta^j_{i,0} \geq 0$ shares of Stock i with $\theta^A_{i,0} + \theta^B_{i,0} = 1$.
- Stock i pays a dividend of $D_{i,T}$ only at the terminal date T, with $D_{1,t}$ being a diffusion process and $D_{2,t}$ being a jump process.
- Agents A and B have heterogeneous beliefs over the expected growth rate of $D_{1,t}$ and the jump intensity of $D_{2,t}$.

The Dividend Processes

- In the view of Agent j, the two processes $D_{1,t}$ and $D_{2,t}$, $t \in [0, T]$, follow:

$$dD_{1,t} = \mu_{1t}^{j} dt + \sigma dZ_t^{j}$$
$$dD_{2,t} = (\mu_2^{j} - \kappa^{j} \mu_J) dt + \mu_J dN_t^{j},$$

  where for $D_{1,t}$, μ_{1t}^{j} is the expected growth rate, and σ is the constant volatility; for $D_{2,t}$, μ_2^{j} is the constant expected growth rate, κ^{j} is the constant jump intensity, and μ_J is the constant jump size.

Equivalent Probability Measures and Stochastic Disagreement

We assume $\mu_{1,t}^B = \mu_{1t}^A + \delta_t$, and $\mu_2^B = \mu_2^A + (\kappa^B - \kappa^A)\mu_J$, where the stochastic disagreement process δ_t follows:

$$d\delta_t = -k(\delta_t - \bar{\delta})dt + \sigma_\delta dZ_t.$$

This implies that the two equivalent probability measures for Agents A and B $\mathbf{P}^A$ and $\mathbf{P}^B$ have a Radon-Nikodym derivative of:

$$\frac{d\mathbf{P}^B}{d\mathbf{P}^A}|_{\mathcal{F}_T} = \eta_T = e^{\int_0^T \frac{\delta_t}{\sigma} dZ_t^A - \int_0^T \frac{\delta_t^2}{\sigma^2} dt} \cdot e^{-(\kappa^B - \kappa^A)T} (\frac{\kappa^B}{\kappa^A})^{N_T^A}.$$

Circuit Breaker and Optimization Problems

- Let $S_{i,t}$ be the price of Stock i at time t.
- Circuit breaker condition: Market is closed until T as soon as $S_{1,t} + S_{2,t} \leq h$, where h is the threshold.
- Each agent maximizes the expected utility:

$$max_{\theta^j_{1,t},\theta^j_{2,t}} \mathbb{E}_t[u(\theta^j_{1,T} D_{1,T} + \theta^j_{2,T} D_{2,T})],$$

  subject to the circuit breaker condition, where

$$u(w) = -\exp(-\gamma w).$$

- In equilibrium, we must have the market clearing condition satisfied, i.e., $\theta^A_{i,t} + \theta^B_{i,t} = 1$ for $i = 1, 2$.

Equilibrium

(1) We first solve each agent's optimization problem at the circuit breaker trigger time τ,

$$\max_{\theta^j_{1,\tau},\theta^j_{2,\tau}} \mathbb{E}^j_\tau[u(W^j_T)],$$

then obtain the market clearing prices $S_{i,\tau}$, the optimal share holdings at τ, and the indirect utility function $V^j(W,\tau)$.
(2) Then solve for the equilibrium prices at $t<\tau$ by solving the planner's problem:

$$\max_{W^A_\nu,W^B_\nu} \mathbb{E}_0[V^A(W^A_\nu,\nu)+\xi\eta_\nu V^B(W^B_\nu,\nu)]$$

s.t. $W^A_\nu + W^B_\nu = S_{1,\nu}+S_{2,\nu}$, where $\nu=\tau\wedge T$.

Stock 1 Price at τ: No price limit

Proposition. (1) For Stock 1, the market clearing price at τ is given by

$$S_1^\tau = D_{1,\tau} + \mu_1^A(T-\tau) - \theta_{1,\tau}^A \gamma\sigma^2(T-\tau)$$

and the optimal shares holding of agent A is

$$\theta_{1,\tau}^A = \frac{-\frac{1}{k}(1-e^{k(t-T)})\delta_t - \bar{\delta}(T-t-\frac{1-e^{k(t-T)}}{k}) + I_\tau}{I_\tau + \gamma\sigma^2(T-\tau)}.$$

Stock 2 price at τ: No price limit

(2) For Stock 2, the market clearing price is given by

$$S_2^\tau = D_{2,\tau} + (\mu_2^A - \kappa^A \mu_J)(T-t) + \sqrt{\kappa^A \kappa^B}\mu_J (T-t) e^{-\frac{\gamma}{2}\mu_J},$$

and the optimal shares holding of stock 2 is

$$\theta_{2,\tau} = \frac{1}{2} - \frac{1}{2\gamma\mu_J}\log(\frac{\kappa^B}{\kappa^A}).$$

Note: $S_2^\tau \equiv \hat{S}_{2,\tau}, \theta_{2,\tau} \equiv \hat{\theta}_{2,\tau}$

The Equilibrium Prices

After obtaining the market clearing prices, we solve the individual indirect utility maximization at τ:

$$V^j(W_\tau^j, \tau) = \max_{\theta_\tau^j} \mathbb{E}_\tau[u(W_\tau^j + \theta_\tau^j(S_T - S\tau)')]$$

Then, we solve the planner's problem as stated before.

State Price Density and Stock Prices

The state price density under agent A's beliefs is

$$\pi_t^A = \mathbb{E}_t^A[\zeta V_1^A(W_\nu^A, \nu)]$$

for some constant ζ. Thus, the stock prices in equilibrium are

$$S_{i,t} = \mathbb{E}_t^A\left[\frac{\pi_\nu^A S_{i,\nu}}{\mathbb{E}_t[\pi_\nu^A]}\right], \quad i = 1, 2.$$

with

$$S_{i,\nu} = \begin{cases} D_{i,T}, & \textit{if } \tau \geq T, \\ S_i^\tau, & \textit{if } \tau < T. \end{cases}$$

Initial Stock Prices

Let $h = (1-\alpha)(S_{1,0} + S_{2,0})$, where α is a given constant. Then the initial stock prices can be found by solving a fixed point problem

$$S_{i,0} = \mathbb{E}_0^A \left[\frac{\pi_\nu^A S_{i,\nu}}{\mathbb{E}_0[\pi_\nu^A]} \right].$$

We numerically solve for the equilibrium prices and illustrate impacts of circuit breakers on the market equilibrium quantitatively.

The Case with Price Limit

When the index jumps beyond the threshold h, the circuit breaker prevents Stock 2 price from dropping to the fundamental level and thus has a strictly positive price limit effect.
In this case, we have Stock 2 price $S_2^\tau = h - S_1^\tau$ and

- The equilibrium share holding for Stock 1 is exactly the same as $\theta_{1,\tau}^j$ for Agent j.
- For Stock 2, the trading between $\tau-$ and τ is dictated by the following rule:
  - If Agents A and B would choose to trade in different directions, then only the smaller trading need is met.
  - If Agents A and B would choose to trade in the same direction, then no one can trade.

Separate Circuit Breakers

- An alternative circuit breaker design: circuit breaker is imposed for each stock separately, i.e., when one stock triggers its threshold, the trading for this stock halts, but trading for other stocks continues.
- Two main benefits:
    1. Separate circuit breakers would not cause correlation or contagion.
    2. The thresholds for different stocks can be set at different levels.

Default Parameter Values for Numerical Analysis

Default parameter values:

$$
\begin{array}{lllll}
\mu_1^A = 0.10/250, & \sigma = 0.06, & \delta_0 = 0.05, & \sigma_\delta = 0.03, & k = 0 \\
\mu_2^A = 0.10/250, & \mu_J = -0.25, & \kappa_2^A = 0.5, & \kappa_2^B = 0.05 & \\
\gamma = 0.1, & \alpha = 0.07. & & &
\end{array}
$$

Contagion

- Crash Contagion: A crash of one stock can cause another stock to crash.
- Positive Correlations: When one stock price changes, the other stock price tends to move in the same direction.
- Volatility Contagion: When the volatility of one stock increases, the volatility of the other stock also increases.

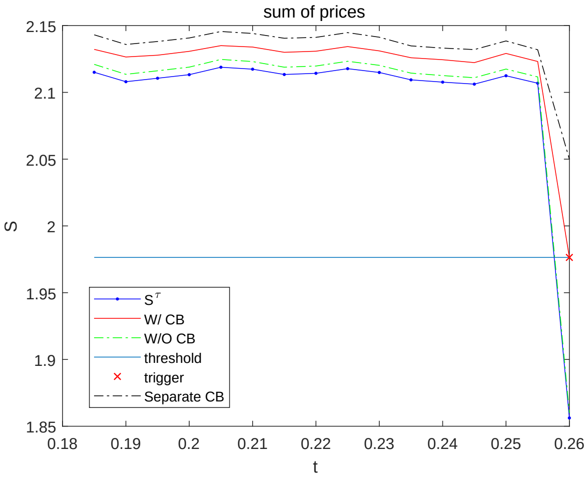

Figure 2: This figure shows a sample of the sum of prices. The market is early halted at the time when the red line touches the threshold at the red cross. In this sample, the breaker is triggered by a jump occurring in S_2. The black dash-dot line is the sum of prices in the presence of individual circuit breakers (One stock halts trading when its price reaches a specified threshold. Another stock keeps trading if its price does not reaches another specified threshold.)

Individual Prices

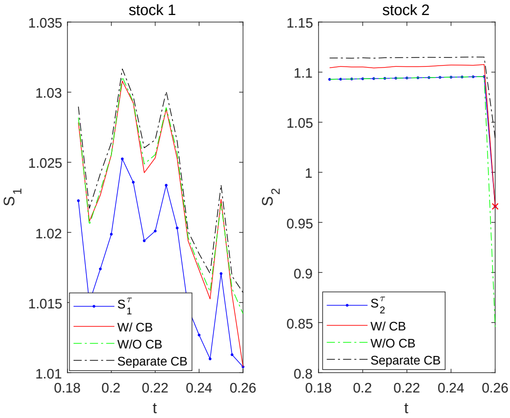

Figure 3: This figure shows the two prices in the sample as Figure 1. The green dot-dash lines are the prices when there is no circuit breaker. The red lines are the prices with a circuit breaker. The blue lines with dots are S_1^τ. In this sample, the breaker is triggered by a jump occurring in the price S_2 (the right panel). [As a result, the price S_1 is dragged down to the equilibrium price at τ: S_1^τ (the left panel), even through the price without circuit breakers goes upward.]

Conditional Distribution

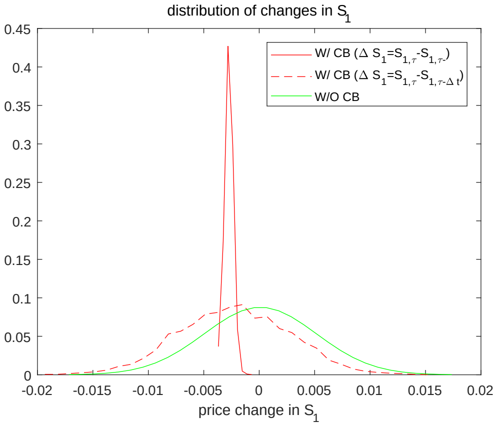

Figure 4: This figure shows distribution of price changes in stock 1 conditional on the circuit breaker is triggered by a jump in Stock 2 price.

Correlations

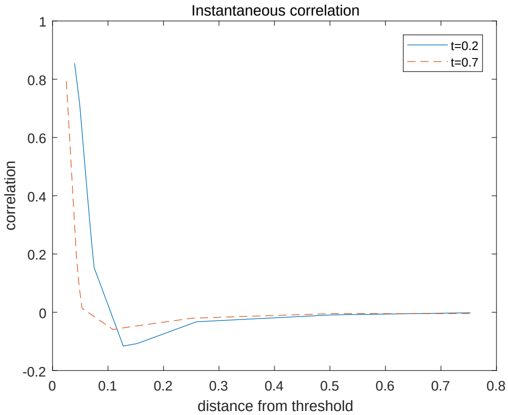

Figure 5: This figure shows that the correlation tends to be higher when the circuit breaker is more likely to be triggered.

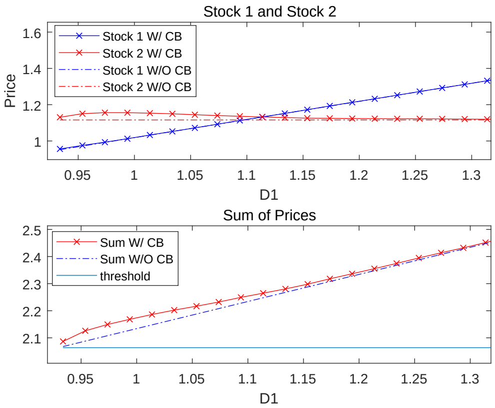

Figure 6: This figure illustrates why the correlation is positive when the threshold is close and why it turns to be positive when the distance is larger. Eventually, $S_{2,t}$ approaches to a constant and the correlation becomes almost zero.

Volatility

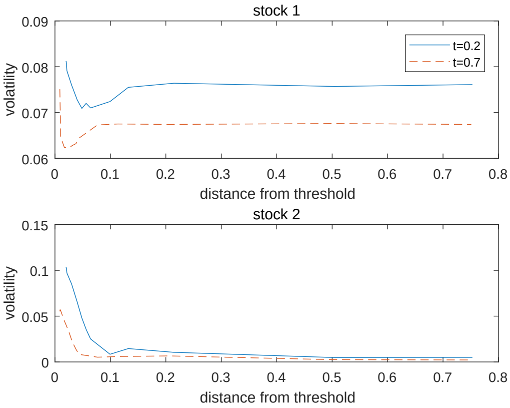

Figure 7: This figure shows that the volatilities tend to be higher when the circuit breaker is more likely to be triggered. This is for $t = 0.1$.

Volatility Contagion

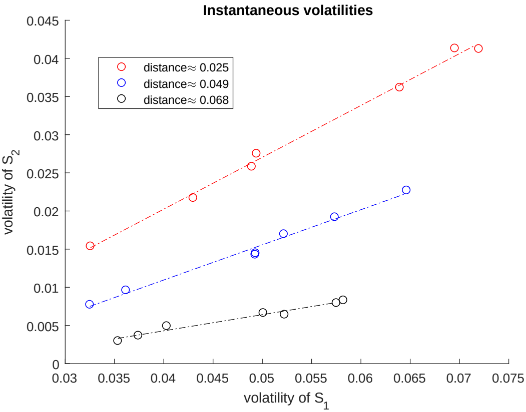

Figure 8: This figure shows how the two volatilities change as σ changes.

Magnet Effect

- Compared to the case without circuit breakers, the index has a higher probability to reach the threshold.
- When the index is closer to the threshold, the falling speed of index is larger.

Probability to Trigger CB

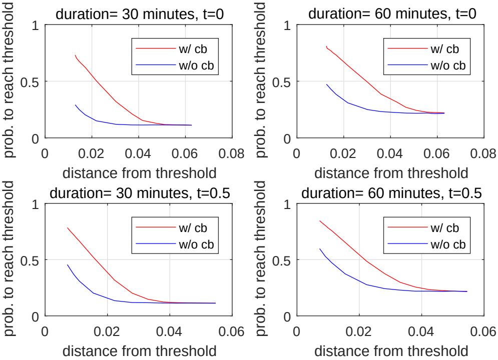

Figure 9: This figure shows probabilities of prices to reach the threshold with or without a circuit breaker.

Average Prices Prior to Market Closure

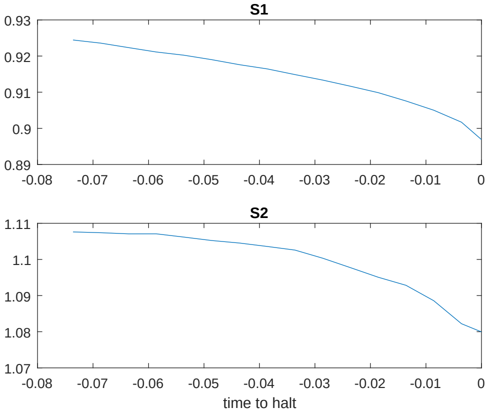

Figure 10: This figure shows the average prices during a short time period right prior to the early closure of market caused by stock 1.

Falling Speed

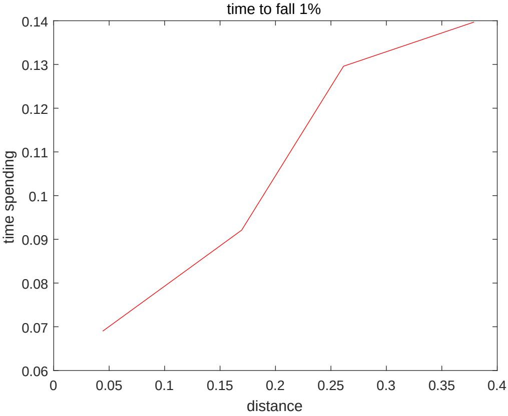

Figure 11: This figure shows the average falling speeds of prices during the short time periods right prior to the early closure of market.

Conclusions

- We develop a continuous-time equilibrium model with circuit breakers on multiple stocks
- We find that in bad times circuit breakers may significantly contribute to
  1. crash and volatility contagion
  2. high correlation among stocks
  3. accelerated market-wide decline
- Separate circuit breakers imposed on individual stocks would reduce the effect on contagion and correlations.