Capital allocation under Fundamental Review of Trading Book

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Fundamental Review of Trading Book (FRTB)

Basel Committee
on Banking Supervision

STANDARDS

Minimum capital
requirements for
market risk

January 2016

Basel 2, 2.5 and FRTB

Basel 2 and 2.5

- 10 days P&L of different risk positions are aggregrated

Basel 2, 2.5 and FRTB

Basel 2 and 2.5

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  Liquidity is not taken into account

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Basel 2 and 2.5

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FRTB sets out revised standards for minimum capital requirements for market risk

- Incorporate the risk of market illiquidity
- An Expected Shortfall (ES) measure
- Constrain the capital-reducing effects of hedging

Structure and implementation

- Standardized approach (SA), Internal models approach (IMA)
- QIS shows that capital charge increases 128% in SA and 54% in IMA (average over 44 banks)
- Model approval down to desk level

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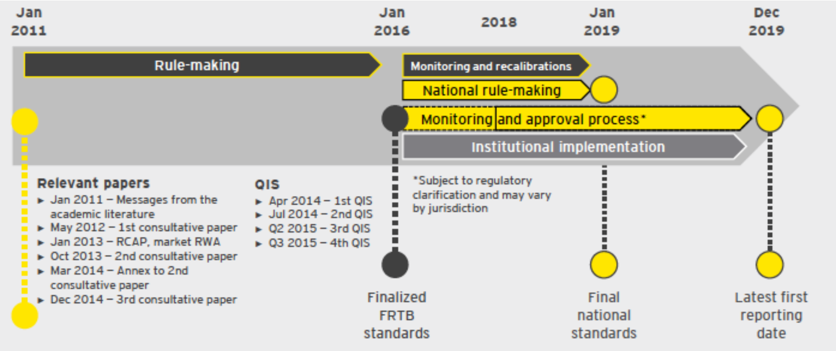
Implementation timeline (Picture from EY)



Impact of FRTB

Consulting firm Oliver Wyman estimates that banks need to spend $5 billion to get ready for FRTB

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Capital charge in SA is very expensive.

IMA requires 90 or more times of calculations than the current rule.

Outline

- FRTB ES and its properties
- Capital allocation
- Two allocation methods under FTRB
- Simulation analysis

Risk factor and liquidity horizon bucketing

P&L of a risk position is attributed to

$$\{\mathrm{RF}_i : 1 \leq i \leq 5\} = \{\mathrm{CM}, \mathrm{CR}, \mathrm{EQ}, \mathrm{FX}, \mathrm{IR}\}$$

$$\{\mathrm{LH}_j : 1 \leq j \leq 5\} = \{10, 20, 40, 60, 120\}$$

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BCBS (2016) 181(k)

Risk factor category	*n*
Interest rate: specified currencies - EUR, USD, GBP, AUD, JPY, SEK, CAD and domestic currency of a bank	10
Interest rate: – unspecified currencies	20
Interest rate: volatility	60
Interest rate: other types	60
Credit spread: sovereign (IG)	20
Credit spread: sovereign (HY)	40
Credit spread: corporate (IG)	40
Credit spread: corporate (HY)	60
Credit spread: volatility	120

Risk factor category	*n*
Equity price (small cap): volatility	60
Equity: other types	60
FX rate: specified currency pairs[37]	10
FX rate: currency pairs	20
FX: volatility	40
FX: other types	40
Energy and carbon emissions trading price	20
Precious metals and non-ferrous metals price	20
Other commodities price	60

Risk profile

Loss: Negative of P&L

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Consider a portfolio of N risk positions. $1 \leq n \leq N$

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$\sum_{i,j} \tilde{X}_n(i,j)$: total loss (over 10 days) of the risk position n

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Liquidity horizon adjusted loss:

$$X_n(i,j) = \sqrt{\frac{\mathrm{LH}_j - \mathrm{LH}_{j-1}}{10}} \sum_{k=j}^{5} \tilde{X}_n(i,k), \quad 1 \leq i,j \leq 5$$

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We record the liquidity horizon bucketing by a 5×5 matrix:

$$X_n = \{X_n(i,j)\}_{1 \leq i,j \leq 5}$$

and call the matrix the risk profile of position n.

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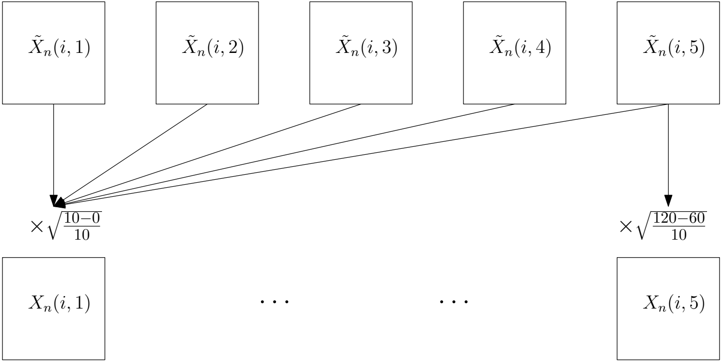
The risk profile of a portfolio is

$$X = \sum_n X_n.$$


$\tilde{X}_n(i,1)$
$\tilde{X}_n(i,2)$
$\tilde{X}_n(i,3)$
$\tilde{X}_n(i,4)$
$\tilde{X}_n(i,5)$
$\times\sqrt{\frac{10-0}{10}}$
$\times\sqrt{\frac{120-60}{10}}$
$X_n(i,1)$
$\cdots$
$\cdots$
$X_n(i,5)$

FRTB ES

The FRTB expected shortfall for portfolio loss attributed to RF_i is

$$\mathrm{ES}(X(i)) = \sqrt{\sum_{j=1}^{5} \mathrm{ES}(X(i,j))^2},$$

where $\mathrm{ES}(X(i,j))$ is the expected shortfall of $X(i,j)$ calculated at the 97.5% quantile.

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Example: Consider a portfolio with only one risk position whose is loss is concentrated on RF_i with $\mathrm{LH}_5 = 120$.

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Example: Consider a portfolio with only one risk position whose is loss is concentrated on RF_i with $\mathrm{LH}_5 = 120$.

$$\tilde{X}(i,j) = 0, \quad j = 1, \ldots, 4, \quad \tilde{X}(i,5) \sim N(0,\sigma^2)$$

Then the ES over 120 days is $\sqrt{120/10}\,\sigma \mathrm{ES}(N(0,1))$.

On the other hand, $X(i,j) = \sqrt{\frac{\mathrm{LH}_j - \mathrm{LH}_{j-1}}{10}}\tilde{X}(i,5)$, $1 \le j \le 5$. Then

$$\mathrm{ES}(X(i)) = \sqrt{\sum_{j=1}^{5} \frac{\mathrm{LH}_j - \mathrm{LH}_{j-1}}{10} \mathrm{ES}(\tilde{X}(i,5))^2} = \sqrt{\frac{120}{10}}\sigma \mathrm{ES}(N(0,1)).$$

Stress period scaling

$\mathrm{ES}^{\mathrm{F,C}}(X(i))$: current 12-month, full set of risk factors

$\mathrm{ES}^{\mathrm{R,C}}(X(i))$: current 12-month, reduced set of risk factors

$\mathrm{ES}^{\mathrm{R,S}}(X(i))$: stress period, reduced set of risk factors

Restriction: $\mathrm{ES}^{\mathrm{R,C}}(X(i)) \geq 75\%\, \mathrm{ES}^{\mathrm{F,C}}(X(i))$.

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FRTB ES capital charge BSBC (2016) 181 (d) :

$$\mathrm{IMCC}(X(i)) = \frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))} \mathrm{ES}^{\mathrm{F,C}}(X(i)), \quad 1 \leq i \leq 5.$$

Capital charge for modellable risk factors

Unconstrained portfolio:

$$X_n(6,j) = \sum_{i=1}^{5} X_n(i,j), \quad X(6,j) = \sum_{n} X_n(6,j).$$

We add $X(6,\dot{})$ as the 6-th row of 5×5 matrix, and call it extended risk profile.

IMCC($X(6)$) is calculated similarly as before.

IMCC: BCBS (2016) 189:

The aggregate capital charge for modellable risk factors is

$$\text{IMCC}(X) = \rho \, \text{IMCC}(X(6)) + (1-\rho) \sum_{i=1}^{5} \text{IMCC}(X(i)),$$

where $\rho = 0.5$.

Properties of IMCC

Proposition

(i) *(Positive homogeneity)* $IMCC(aX) = a\, IMCC(X), a \geq 0.$

(ii) *(Sub-additivity for ES) If* $ES((X+Y)(i,j)) \geq 0$, *then*

$$ES((X+Y)(i)) \leq ES(X(i)) + ES(Y(i)).$$

(iii) *(Sub-additivity for IMCC) If*

$$\frac{ES^{R,S}((X+Y)(i))}{ES^{R,C}((X+Y)(i))} \leq \min\left\{\frac{ES^{R,S}(X(i))}{ES^{R,C}(X(i))}, \frac{ES^{R,S}(Y(i))}{ES^{R,C}(Y(i))}\right\},$$

and $ES^{F,C}((X+Y)(i,j)) \geq 0$, *then*

$$IMCC((X+Y)(i)) \leq IMCC(X(i)) + IMCC(Y(i)).$$

Profit and sub-additivity

Example:
Consider X and Y concentrating on RF_i and LH_j.

$$\mathbb{P}(X(i,j) = -1) = \mathbb{P}(X(i,j) = 0) = 0.5, \quad (X+Y)(i,j) \equiv -1.$$

Then $\text{ES}(X(i)) = \text{ES}(Y(i)) = 0$, but $\text{ES}((X+Y)(i,j)) \geq 0$ is violated,

$$\text{ES}((X+Y)(i)) = \left|\text{ES}((X+Y)(i,j))\right| = |-1| = 1 > \text{ES}(X(i)) + \text{ES}(Y(i)).$$

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We propose to floor each $\text{ES}(X(i,j))$ at zero (not required by FRTB)

$$\text{ES}^+(X(i)) = \sqrt{\sum_{j=1}^{5} \text{ES}^+(X(i,j))^2},$$

where $\text{ES}^+(X(i,j)) = \max\{\text{ES}(X(i,j)), 0\}$.

The resulting FRTB ES is sub-additivity and positive homogeneous.

Capital allocation

Consider a portfolio of N risk positions with losses $L_1, \ldots, L_N$.

The total loss is $L = \sum_{n=1}^{N} L_n$.

ρ is a risk measure.

An allocation is a map $\mathrm{Law}(L_1, \ldots, L_N) \to \mathbb{R}^N$:

$$\mathrm{Law}(L_1, \ldots, L_N) \mapsto \rho(L_n \mid L), \qquad \text{for each } n,$$

such that

$$\sum_{n=1}^{N} \rho(L_n \mid L) = \rho(L).$$

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Banks need allocations to calculate return on risk-adjusted capital (RORAC):

$$\frac{-\mathbb{E}[L_n]}{\rho(L_n \mid L)}.$$

RORAC evaluates the capital efficiency of each position.

Euler allocation principle

Let $v_1, \ldots, v_N$ be a sequence of numbers and $L^v = \sum_{i=1}^N v_n L_n$.

Per-unit Euler allocation is

$$\rho(L_n \mid L)(v) := \frac{\partial}{\partial v_n} \rho(L^v).$$

Setting all $v_n = 1$, we denote the allocation to L_n as $\rho(L_n \mid L)$.

if ρ is homogeneous of degree 1, Euler's theorem for homogeneous functions implies

$$\rho(L^v) = \sum_n v_n \frac{\partial}{\partial v_n} \rho(L^v).$$

Setting $v = 1$, we have the full allocation property.

Pros and Cons of Euler allocation

Tasche (1999) shows that

$$\frac{\partial}{\partial v_n}\left(\frac{-\mathbb{E}[L^v]}{\rho(L^v)}\right)\begin{cases} >0, \text{ if } \frac{-\mathbb{E}[L_i]}{\rho(L_i \mid L)(v)} > \frac{-\mathbb{E}[L^v]}{\rho(L^v)} \\ <0, \text{ if } \frac{-\mathbb{E}[L_i]}{\rho(L_i \mid L)(v)} < \frac{-\mathbb{E}[L^v]}{\rho(L^v)} \end{cases}$$

Denault (2001) uses corporative game (Shapley (1953) and Aumann-Shapley (74)) to show that the Euler allocation is the "fair" allocation.

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However,

- Euler allocation is unstable.
- Euler allocation induces large negative allocations.

Two steps allocation for FRTB

Given liquidity horizon adjusted risk profiles $\{X_n\}_{1 \le n \le N}$,

Step 1: allocate to each $X_n(i,j)$

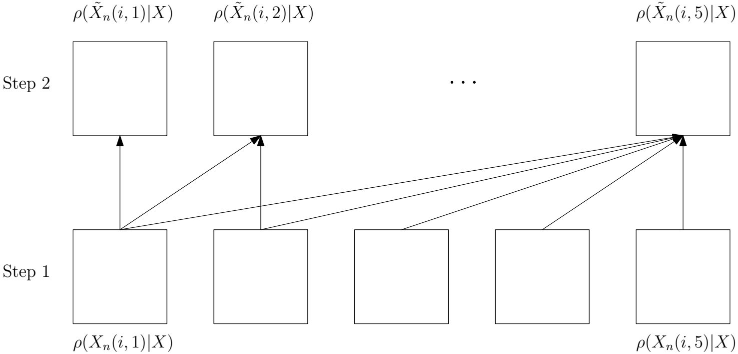
$$\rho(X_n(i,j) \,|\, X).$$

Step 2: allocate to each $\tilde{X}_n(i,k)$

$$\rho\big(\tilde{X}_n(i,k) \,|\, X_n(i,j)\big), \quad k \ge j.$$

Then aggregate

$$\rho(\tilde{X}_n(i,k) \,|\, X) = \sum_{j=1}^{k} \rho(X_n(i,k) | X_n(i,j)).$$



$\rho(\tilde{X}_n(i,1)|X)$
$\rho(\tilde{X}_n(i,2)|X)$
$\rho(\tilde{X}_n(i,5)|X)$
Step 2
. . .
Step 1
$\rho(X_n(i,1)|X)$
$\rho(X_n(i,5)|X)$

Euler allocation under FRTB

Let $v = \{v_1, \dots, v_n\}$ be real numbers

Let $X^{v,j}(i) = \sum_n X_n^{v_n,j}(i)$, where

$$X_n^{v_n,j}(i) = \big(X_n(i,1), \cdots, X_n(i,j-1), v_n X_n(i,j), X_n(i,j+1), \cdots, X_n(i,5)\big).$$

For each RF_i, we define the Euler allocation for FRTB ES as

$$\mathrm{ES}(X_n(i,j) \,|\, X(i)) := \frac{\partial}{\partial v_n} \mathrm{ES}(X^{v,j}(i))\Big|_{v=1},$$

where $v = 1$ means all $v_n = 1$.

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Lemma

$$ES(X_n(i,j)\,|\,X(i)) = \frac{ES(X(i,j))}{ES(X(i))}\,\frac{\partial}{\partial v_n}ES\big(X^v(i,j)\big)\Big|_{v=1},$$

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$$\frac{\partial}{\partial v_n} \mathrm{ES}\big(X^v(i,j)\big)\Big|_{v=1} = \mathbb{E}\big[X_n(i,j) \,|\, X(i,j) \geq \mathrm{VaR}(X(i,j))\big].$$

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- If each $X(i,j)$ is floored at zero, then

$$\text{ES}^+(X_n(i,j)\,|\,X(i)) = \begin{cases} \frac{\text{ES}^+(X(i,j))}{\text{ES}^+(X(i))}\,\mathbb{E}\big[X_n(i,j)\,|\,X(i,j) \geq \text{VaR}(X(i,j))\big] & \text{if ES}(X(i,j)) > 0 \\ 0 & \text{otherwise} \end{cases}.$$

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- Euler allocation of IMCC

$$\mathrm{IMCC}^E(X_n(i,j) \,|\, X) := 0.5 \, \frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))} \, \mathrm{ES}^{\mathrm{F,C}}(X_n(i,j) \,|\, X(i)).$$

It is a full allocation.

Negative allocations

Hedging among different RFs or LHs does not lead to negative allocations.

Example:

Consider two loss Y with RF_i and Z with RF_k, $i \neq k$, $Y, Z \sim N(0, \sigma)$, and $Y + Z = 0$.

Euler of regular ES:

$$\text{ES}^R(Y|Y+Z) + \text{ES}^R(Z|Y+Z) = \text{ES}^R(Y+Z) = 0.$$

Then one of two allocations must be negative, say $\text{ES}^R(Y|Y+Z)$.

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Euler of FRTB ES:
Let X be the risk profile containing X and Y. $X(i) = Y$, $X(k) = Z$, then

$$\text{ES}(Y|X(i)) = \text{ES}(Y) > 0, \quad \text{ES}(Z|X(k)) = \text{ES}(Z) > 0.$$

Even though $Y + Z = 0$, $IMCC(X) > 0$.

Constrained Aumann-Shapley allocation

Motivated by Li, Naldi, Nisen, and Shi (2016), who combine Shapley and Aumann-Shapley allocations.

LH permutation matrix

$$\mathcal{L} := \begin{bmatrix} 10 & 20 & 40 & 60 & 120 \\ 10 & 20 & 40 & 120 & 60 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 120 & 60 & 40 & 20 & 10 \end{bmatrix}_{5! \times 5} .$$

Let $\mathcal{L}^{-1}(r, j)$ be the column of $\mathcal{L}$ in which LH_j locates. e.g. $\mathcal{L}^{-1}(2, 5) = 4$.

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Let $v = \{v_1, \dots, v_N\}$,

$$X^{v,r,j}(i) = \sum_n X_n^{v,r,j}(i),$$

where $X^{v,r,j}(i)$ is a row depending on when LH_j appears in r.

For example,

$$X_n^{v,2,5}(i) = \big(X_n(i,1), X_n(i,2), X_n(i,3), 0, v_n X_n(i,5)\big).$$

Constrained Aumann-Shapley allocation

We define the Constrained Aumann-Shapley allocation(CAS) in the permutation r as

$$\mathrm{CAS}(r, X_n(i,j)) := \int_0^1 \frac{\partial}{\partial v_n} \mathrm{ES}(X^{v,r,j}(i))\Big|_{v=q} dq,$$

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Lemma

$$CAS(r, X_n(i,j)) = \eta(r,i,j) \frac{\partial}{\partial v_n} ES\big(X^v(i,j)\big)\Big|_{v=1},$$

where

$$\eta(r,i,j) = \frac{\sqrt{\sum_{1\le s\le \mathcal{L}^{-1}(r,j)} ES\big(X(i,\mathcal{L}(r,s))\big)^2} - \sqrt{\sum_{1\le s< \mathcal{L}^{-1}(r,j)} ES\big(X(i,\mathcal{L}(r,s))\big)^2}}{ES\big(X(i,j)\big)}.$$

- CAS is a scaled version of Euler
- Losses with the same LH need to be added to the same portfolio at the same time, to ensure computational efficiency

CAS allocation of IMCC

We define the CAS allocation for IMCC as

$$\mathrm{IMCC}^C(X_n(i,j)\,|\,X) := 0.5\,\frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))}\,\frac{1}{5!}\sum_{r=1}^{5!}\mathrm{CAS}^{\mathrm{F,C}}(r, X_n(i,j)).$$

It is a full allocation.

Stress scaling adjustment

In the previous two methods, the $X_n(i,j)$ induced risk contribution is not considered in the stress scaling factor $\frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))}$.

We define the Euler allocation with stress scaling adjustment as

$$\mathrm{IMCC}^{\mathrm{E,S}}(X_n(i,j)\,|\,X(i)) := 0.5\frac{\partial}{\partial v_n}\left[\frac{\mathrm{ES}^{\mathrm{R,S}}(X^{v,j}(i))}{\mathrm{ES}^{\mathrm{R,C}}(X^{v,j}(i))}\mathrm{ES}^{\mathrm{F,C}}(X^{v,j}(i))\right]\Big|_{v=1}.$$

Lemma

$$IMCC^{E,S}(X_n(i,j)\,|\,X(i)) = 0.5\Big[\frac{ES^{R,S}(X(i))}{ES^{R,C}(X(i))}\,ES^{F,C}(X_n(i,j)\,|\,X(i)) + \frac{ES^{F,C}(X(i))}{ES^{R,C}(X(i))}\,ES^{R,S}(X_n(i,j)\,|\,X(i)) - \frac{ES^{R,S}(X(i))ES^{F,C}(X(i))}{ES^{R,C}(X(i))^2}\,ES^{R,C}(X_n(i,j)\,|\,X(i))\Big].$$

Simulation analysis 1

$\tilde{X}(i, j)$, normal with mean 0 and annual volatility 30%

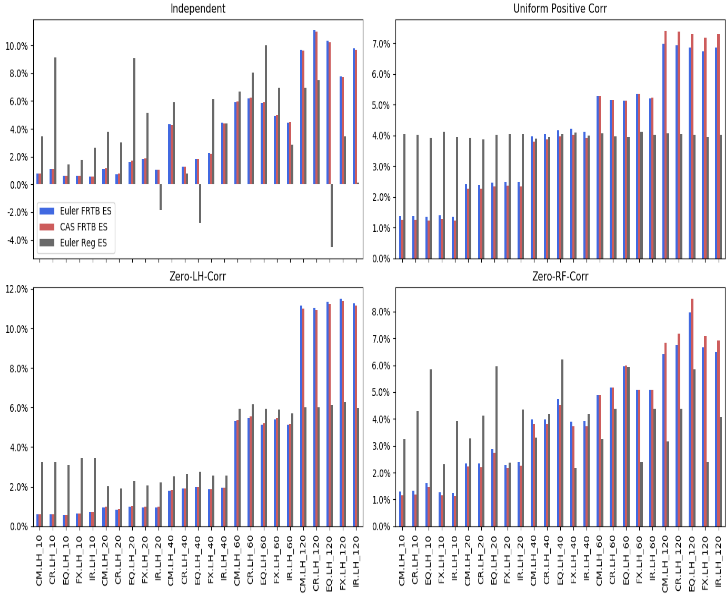
Risk profiles are simulated for 250 days

Independence among different days

The following correlation structure in the same day:

1. Independence
2. Strong positive correlation among RFs and LHs
3. Strong positive correlation among RFs, independent among LHs
4. Independent among RFs, strong positive correlation among LHs

Longer LH leads to larger percentage of allocation



Simulation analysis 2

Three hedging structures:

1. Strong hedging between EQ and IR
2. Strong hedging between LH_1 and LH_2
3. Strong hedging between two risk positions in the same bucket

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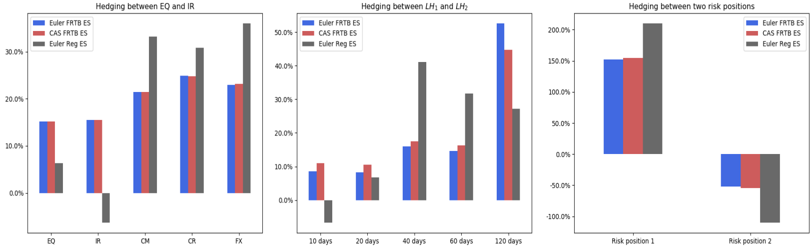
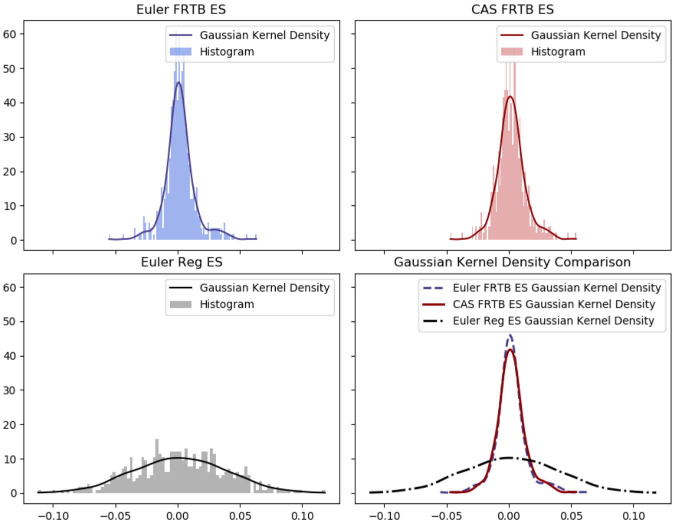


FRTB allocations produce

- no negative allocations for hedging among different bucket
- some negative allocations for hedging in the same bucket, but with smaller magnitude

FRTB allocations are more stable

Histograms and fitted kernel densities for allocations in Case 3:



Simulation analysis 3

Loss in EQ with 40-days LH and CM with 60 days LH have 9 times of volatility in the stress period than the normal period.

Two reduced set of risk factors

Set A : Include both RFs with large variations

Set B : Exclude both RFs with large variations

Simulation analysis 3

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	Set A (Adj)	Set A (Without adj)	Set B (Adj)	Set B (Without adj)
CM.60	4.00%	2.24%	1.43%	1.43%
EQ.40	5.04%	3.26%	2.11%	2.11%

Table: IMCC(Set A)=11.55 and IMCC(Set B)=3.14

The choice of reduced set of risk factors has large impact on allocations.

Conclusion

Two allocation methods reduce FRTB allocations to Euler allocations

- Computational efficiency
- Easy to adapt to the current system

Simulation analysis shows

- Longer LH leads to more allocation
- Much less negative allocations
- More stable allocations
- Sensitive to the choice of reduced set of risk factors

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Thanks for your attention!