Dynamic Valuation and Hedging of a Book

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Outline

- Defining the Book

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- Unhedged Valuation

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- Explicit Driver Constructions

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- Maximum Likelihood Estimation for Spot Average Ratio Dependence
- Markovian model using Average and Spot Average Ratio
- Implementation of Hedge for a Sample Book

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- The functions may be constructed from a knowledge of positions and we presume access to such functions.
- They define the book to be valued and hedged.

Hedged Book Outcomes

- For stock positions in dollar terms of $a(S(u), u)$ at time u and discount rate r the present value of such flows to date are

$$\sum_{t_i \le t} c_i \left(S(t_i), t_i\right) e^{-rt_i} + \int_0^t \int_{-\infty}^{\infty} e^{-ru} a(S(u\_), u) \left(e^x - 1\right) \mu(dx, du)$$

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- where $\mu(dx, du)$ is the integer valued random measure associated with the finite variation jumps in the log price relative of the stock price.

Unhedged Valuation

- For valuation we follow the $\mathcal{G}-$ *expectations* approach introduced in Peng (2006) with valuations defined by nonlinear expectations that are unique viscosity solutions to equations of the form

$$V_t = \mathcal{G}(V) - rV$$

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$$\begin{aligned} V(S,0) &= c_T(S) \\ V(S,t_i) &= V(S,t_{i\_}) + c_i(S),\ 0 < t_1 < \cdots < t_N = T. \end{aligned}$$

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- The result is a nonlinear valuation at time t of the future cash flows yet to be realized, when the spot at time t is at level S.

Spatially Inhomogeneous Compensator

- The operator is defined in terms of measure distortions that distort the compensator of μ that we take to possibly be dependent on the level of S and in the bilateral gamma class.

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- We write

$$\nu(S,x)dxdt = \begin{pmatrix} \frac{c_p(S)}{x}\exp\left(-\frac{x}{b_p(S)}\right)\mathbf{1}_{x>0} \\ +\frac{c_n(S)}{|x|}\exp\left(-\frac{|x|}{b_n(S)}\right)\mathbf{1}_{x<0} \end{pmatrix} dxdt$$

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- The spatially inhomogeneous compensating measure is then

$$\nu(S,A) = \int_A \nu\left(S,x\right)dx$$

One Price Issues

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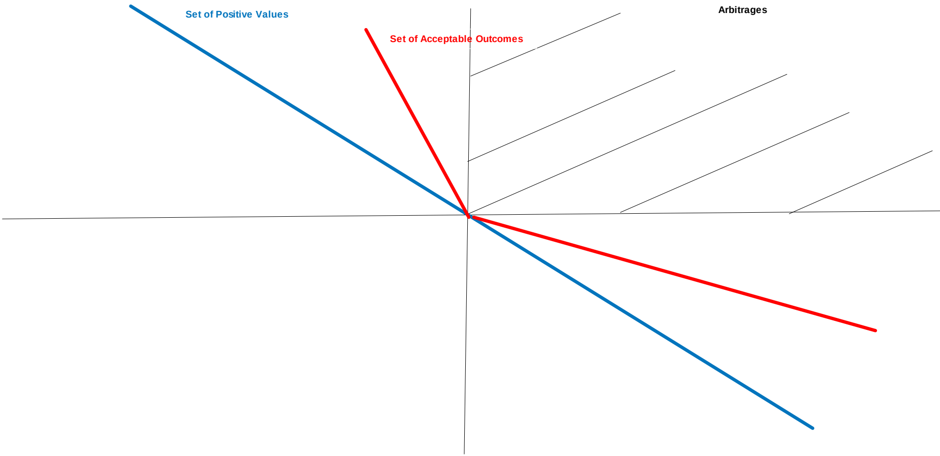
- Hence the valuation function is linear.
- The set of acceptable risks are those with a positive value and this is a very large convex set.
- Useful as it may be, it renders optimization useless and defines risk acceptability too generously.

Graph of Arbitrages, Positive Alpha, and Acceptable Opportunities



Conservative valuation in two price economies

- When risk acceptability is contracted from a half space to a proper convex cone containing the nonnegative outcomes that are certainly acceptable at zero cost, then the acceptable risks become all outcomes X satisfying

$$E^Q[X] \geq 0, \text{ for all } Q \in \mathcal{M}$$

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- Selling X at $b(X)$ or buying it for $a(X)$ requires that

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- are acceptable.
- The best and bid and ask prices are

$$b(X) = \inf_{Q \in \mathcal{M}} E^Q[X]$$
$$a(X) = \sup_{Q \in \mathcal{M}} E^Q[X]$$

Further Remarks on Conservative Valuation I

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- We now ask why one should scale value with the scale of the outcome?
- If the value is market based and contemplated positions are small relative to the size of the market then the value of twice the outcome should be twice the value.

Conservative valuation and probability distortions

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- Let $\Psi(u)$ be a concave distribution function on the unit interval and evaluate the value of outcome X with distribution function $F(x)$ and density $f(x)$ as

$$b(X) = \int x\Psi'\left(F(x)\right) f(x) dx$$

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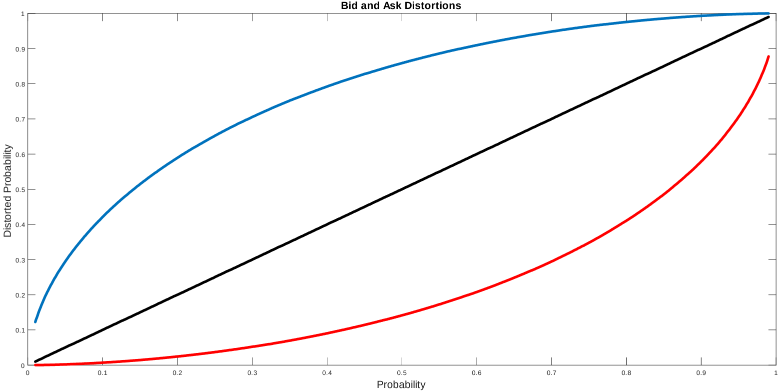
- $\Psi'(u)$ for u near zero lifts the weighting on losses while $\Psi'(u)$ for u near unity reduces the weighting on gains.

Distorted Expectations and Lower Price Operators

- The lower price formed from distorted expectations is associated with $\mathcal{M}$ the set of supporting measures being all measures Q such that

$$Q(A) \leq \Psi\left(P(A)\right), \text{ for all } A.$$

Bid Ask Distortions



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- All outcomes are bounded away from zero on sets of finite measure.
- On sets of infinite measure outcomes converge towards zero reflecting the view that only nothing happens all the time.

Measure Distorted Valuation

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- The measure distorted lower value of outcome X on an infinite measure space with measure μ is defined by

$$b(X) = -\int_0^\infty G^+\left(\mu\left(X < -a\right)\right)da + \int_0^\infty G^-\left(\mu\left(X > a\right)\right)da$$

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- The upper value is given by

$$a(X) = -\int_0^\infty G^-\left(\mu\left(X < -a\right)\right) da + \int_0^\infty G^+\left(\mu\left(X > a\right)\right) da$$

Measure Distortions and Lower Valuations

- The lower valuation using measure distortions is

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\begin{aligned}
b(X) &= \inf_{\widetilde{\nu}\in\mathcal{M}} \int X(\omega)\widetilde{\nu}(d\omega) \\
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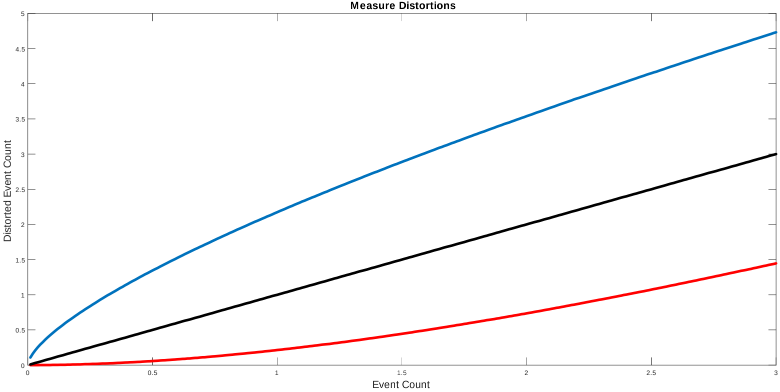
- where $\widetilde{\nu} \in \mathcal{M}$ just if

$$G^{-}(\nu(A)) \leq \widetilde{\nu}(A) \leq G^{+}(\nu(A)), \text{ for all } A \text{ with } \nu(A) < \infty.$$

Measure Distortions



Hedged Valuation

- For two measure increasing distortions G^+, G^- concave/convex, above/below the identity and hedge policy $a(S,t)$ we define the operator

$$
\begin{aligned}
\mathcal{G}(V) &= -\int_0^\infty G^+\left(\nu(S, \left[\begin{array}{c} a(S,t)\left(e^x-1\right) \\ +V(Se^x,t)-V(S,t) \end{array}\right]^- > w\right) dw \\
&\quad + \int_0^\infty G^-\left(\nu(S, \left[\begin{array}{c} a(S,t)\left(e^x-1\right) \\ +V(Se^x,t)-V(S,t) \end{array}\right]^+ > w\right) dw
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- The hedge selection defines

$$a(S,t) = \arg\max_a \left(\begin{array}{c} -\int_0^\infty G^+\left(\nu(S, \left[\begin{array}{c} a\,(e^x-1) \\ +V(Se^x,t) - V(S,t) \end{array}\right]^- > w\right) dw \\ +\int_0^\infty G^-\left(\nu(S, \left[\begin{array}{c} a\,(e^x-1) \\ +V(Se^x,t) - V(S,t) \end{array}\right]^+ > w\right) dw \end{array}\right).$$

Connections to BSPIDE's

- The nonlinear valuation may be related to the solution of Backward Stochastic Partial Integro-Differential Equations.

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- with

$$\begin{aligned} X(t) &= \int_{(0,t]\times\mathbb{R}^k\setminus\{0\}} \left((\exp\circ I)(x) - \mathbf{1}_k\right)\nu(X(s), s, x)dxds \\ &\quad + \int_{(0,t]\times\mathbb{R}^k\setminus\{0\}} \left((\exp\circ I)(x) - \mathbf{1}_k\right)\widetilde{N}(dx \times ds) \end{aligned}$$

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- where $(\exp\circ I)(x) = (e^{x_1}, \cdots, e^{x_k})^T$, $\mathbf{1}_k$ is the k dimensional vector with all entries unity, and $\widetilde{N}(dx \times ds)$ is a compensated jump measure

BSPIDE

- On this probability space let χ be a terminal random variable and let B_t be a lower prudential valuation for χ asscoiated with a driver function $g(t,z)$ defined on $(0,T] \times \mathcal{L}^2(\nu(X(t),t,x)dx)$ that is positive, homogeneous and convex in z.

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- The Backward Stochastic Partial Integro-Differential Equation $(BSPIDE)$ solves for (B_t, Z_t) for $t < T$ the equation

$$B_t + \int_t^T g(s, Z_s)ds + \int_{(0,t]\times\mathbb{R}^k\setminus\{0\}} Z_s(x)\widetilde{N}(ds \times dx) = \chi.$$

BSPIDE and Valuation

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$$V(T, X(T)) = \chi$$
- and V solves the semilinear partial integro-differential equation

$$\begin{aligned}
& \dot{V} + \mathcal{K}V(t,x) - g(t, \mathcal{D}V_{t,x}) \\
= \; & 0 \\
& \mathcal{K}V(t,x) = d_t^T \nabla V \\
& + \int_{\mathbb{R}^k \backslash \{0\}} \left(\mathcal{D}V_{t,x} - \nabla V(t,x)^T x \left(e^y - 1 \right) \right) \nu(dy) \\
& \mathcal{D}V_{t,x} = V(t, xe^y) - V(t,x) \\
d_t \; = \; & \int_{\mathbb{R}^k \backslash \{0\}} x \left(e^y - 1 \right) \nu(x, t, dy)
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$$\begin{aligned} \xi &= \frac{dQ}{dP} \\ \xi &= \mathcal{E}(M), \ \mathcal{E} \text{ is stochastic exponential} \\ M &= \int_{(0,T]\times\mathbb{R}^k\backslash\{0\}} H_s(y)\widetilde{N}(ds \times dy) \end{aligned}$$

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- with

$$\int z(y) H_s(y) \nu(X(s), s, y) dy \leq g(s, z), \ z \in \mathcal{L}^2 \left(\nu(X(s), s, y) dy \right).$$

Discounted Valuations

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$$e^{-rt}V(X(t),t) = \inf_{Q\in\mathcal{M}} E^{Q}\left[e^{-rT}\chi\right].$$

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- The variation on the other hand is

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- The semilinear equation in W is on noting $\mathcal{K}W = \mathcal{V}W$ that

$$\dot{W} + \mathcal{K}W - g(t, \mathcal{D}W_{t,x}) = 0$$

BSPIDE and Risk Acceptability

- We now write that

$$e^{-rt}V(t,x) + \int_t^T g(s,Z_s)ds + \int_{(t,T]\times\mathbb{R}^k\setminus\{0\}} Z_s(y)\widetilde{N}(ds\times dy) = \chi.$$

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- Furthermore we have that

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- We consider instead a dependence on the ratio of the index level to a geometrically weighted average of past prices.

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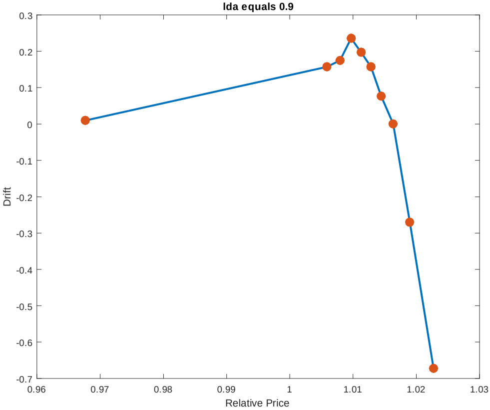
- From the estimated parameters one may evaluate the return drift separately in each bucket.

Drift and the Ratio

- The Figure presents the dependence observed for the drift as a function of the ratio.



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- This avoids having to describe up front a functional form for the dependence.

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- For arbitrary Z levels, we use the closest Z value on the fine grid where the density has been precomputed.
- This minimizes the number of Fourier inversions to be computed.
- The logarithm of the thousand densities evaluated at the observed returns are summed to construct the log likelihood to be maximized.

Parameter Dependence on Spot Average Ratio

- The Figure presents the dependence of the four bilateral gamma parameters on the spot average ratio.

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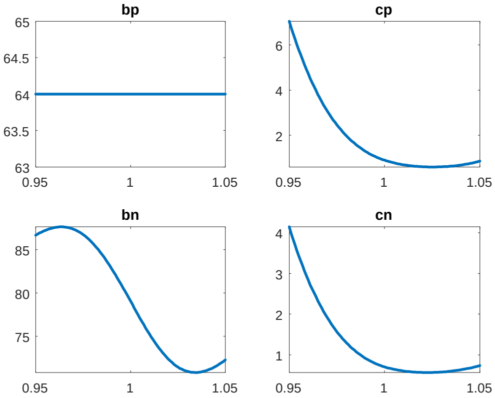

Dependence of Expected Return on Spot Average Ratio

- The Figure show the dependence of the expected return in basis points on the level of the spot average ratio.

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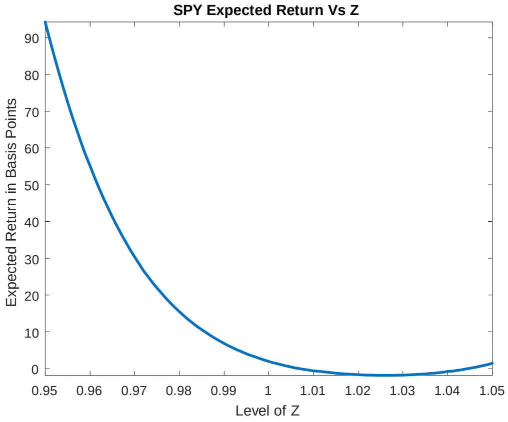

Markov Model for the Average, Y, and the Spot Average Ratio, Z

- In a continuous formulation the average $Y(t)$ may be constructed from the Spot prices $S(t)$ by

$$Y(t) = \theta \int_{-\infty}^{t} e^{-\theta(t-u)} S(u) du.$$

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- The dynamics for the ratio are given by

$$dZ = -\theta Z(t) \left(Z(t) - 1\right) dt Z(t) \left(\left(e^{x} - 1\right) * \mu(dx, du)\right)$$

The Compensator for Z

- The compensator for Z is given by

$$\nu\left(Z,dx\right)dt=\left(\begin{array}{c}\frac{c_p(Z)}{x}\exp\left(-\frac{x}{b_p(Z)}\right)\mathbf{1}_{x>0}\\ +\frac{c_n(Z)}{|x|}\exp\left(-\frac{|x|}{b_n(Z)}\right)\end{array}\right)dxdt$$

Value and Hedge Policy Functions

- Hence $Z(t), Y(t)$ is a two dimensional Markov process and we seek the value and hedge policy functions

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- We do this on an 18 core Alienware Machine.

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- We solve

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- The backward recursion is implemented on 20 time steps.

Integrating the compensator

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- These measures are distorted and integrated over tail levels to construct the measure distorted value.
- Values and Hedge positions are smoothed at each time step using Gaussian Process Regression.

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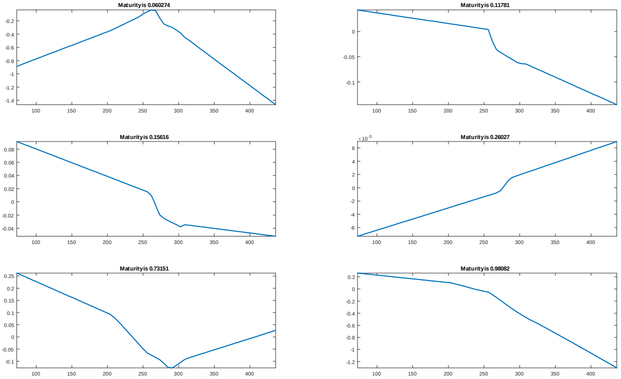
- Summing the spot dependent option payoffs times positions delivers the functions $c_i(S)$.
- There were 29 maturities below a year in the option book ranging from one to 358 days.

Sample Claims By Maturity

- The Figure presents graphs of state contingent claims in millions of dollars for a sample of the 29 maturities



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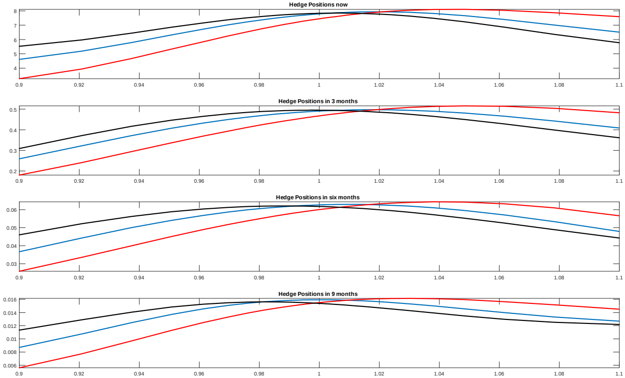
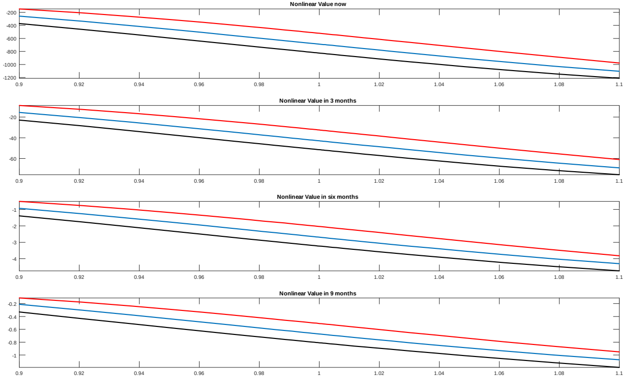


Value Function By Z and Y

- We present four graphs showing the nonlinear value as a function of Z for three settings on Y for now, three, six, and nine months in.



Hedge Position Regression Coefficients

- The hedge positions were regressed on the delta, gamma, and levels of Z, Y, Z^2, Y^2 separately for each level of time maturity.

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- The Table shows results for a sample of times to maturity.

			Regressing Hedge Positions on				
Time Left	Delta	Gamma	Z	Y	Z^2	Y^2	RSQ
1	1.1293	29.9751	1723.6023	195.7961	-8.3502	-0.3197	0.9302
0.9	1.0964	36.8843	318.8334	48.4325	-1.5291	-0.0785	0.9243
0.8	1.4404	40.2417	95.7524	12.5620	-0.4641	-0.0205	0.9200
0.7	2.1472	66.0671	19.6692	3.4710	-0.0950	-0.0057	0.9080
0.6	2.0326	62.7417	8.7496	1.6764	-0.0425	-0.0027	0.9007
0.5	2.3538	73.7335	4.8765	0.8572	-0.0237	-0.0014	0.8791
0.4	5.6210	191.1055	2.6347	0.4515	-0.0129	-0.0007	0.8611
0.3	6.9205	197.6278	3.2125	0.4428	-0.0158	-0.0007	0.8342

Hedge Position T-Stats

- The Table shows the corresponding t-stats.

			T-Stats			
Time Left	Delta	Gamma	Z	Y	Z^2	Y^2
1	17.77	10.68	16.77	73.64	-16.24	-71.09
0.9	15.46	11.71	11.67	68.52	-11.20	-65.67
0.8	18.08	11.29	13.40	67.87	-12.99	-65.32
0.7	22.23	15.23	9.77	66.50	-9.43	-64.38
0.6	19.86	13.64	8.75	64.65	-8.50	-62.60
0.5	19.28	13.47	8.65	58.71	-8.42	-56.99
0.4	18.96	14.29	8.36	55.32	-8.17	-53.76
0.3	21.73	13.70	9.45	50.24	-9.32	-49.09

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