Abstract: In this talk, we first present basic notions and the motivation of Terry Lyons's theory of rough paths. Next, we introduce path derivatives for controlled rough paths in the spirit of Dupire's functional Ito calculus. This allows us to study, in a very convenient manner, rough differential equations with time-dependent coefficients under minimal regularity assumptions with respect to time. Consequently, we can establish existence and stability of pathwise solutions for a large class of stochastic differential equations on a universal canonical sample space. These results are useful to study fully nonlinear stochastic PDEs. If time remains, we present a notion of pathwise viscosity solutions for those equations. Parts of this talk are based on joint works with Rainer Buckdahn, Jin Ma, and Jianfeng Zhang.