A Set-Valued Markov Chain Approach to Credit Default

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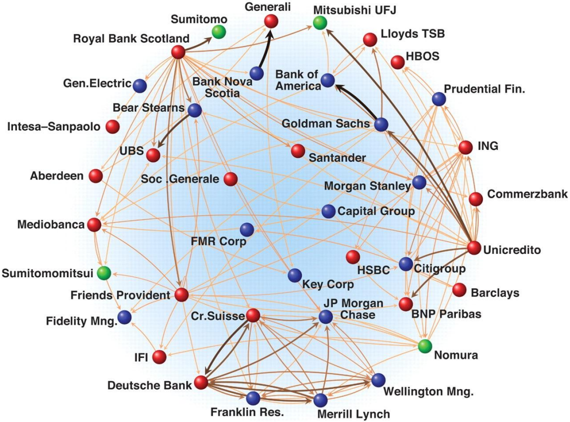
Mathematical Finance Colloquium, USC
March 9, 2020

Highlights

- General credit default model:
  consider *intergroup contagion* and *macroeconomic factors*
- A set-valued Markov chain (MC) for the default process X
  show such an MC exists under given conditions
- Explicit pricing formulas of CDO spreads
- Empirical studies to showcase the theoretical results

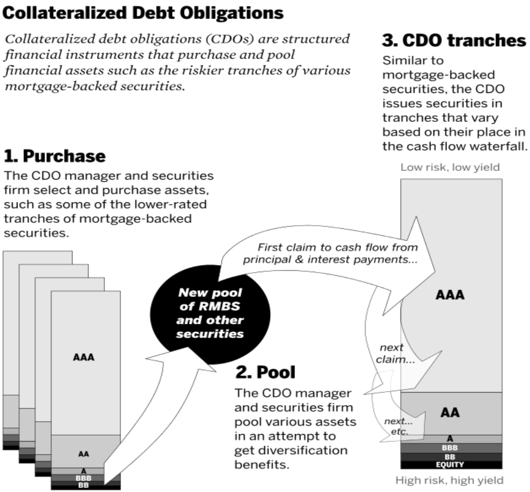
OUTLINE

MOTIVATION



Sample of the international financial network
Credits: ETH Zurich

Motivation


Collateralized Debt Obligations
Collateralized debt obligations (CDOs) are structured financial instruments that purchase and pool financial assets such as the riskier tranches of various mortgage-backed securities.
1. Purchase
The CDO manager and securities firm select and purchase assets, such as some of the lower-rated tranches of mortgage-backed securities.
AAA
AA
A
BBB
BB
New pool of RMBS and other securities
2. Pool
The CDO manager and securities firm pool various assets in an attempt to get diversification benefits.
First claim to cash flow from principal & interest payments...
next claim...
next... etc.
3. CDO tranches
Similar to mortgage-backed securities, the CDO issues securities in tranches that vary based on their place in the cash flow waterfall.
Low risk, low yield
AAA
AA
A
BBB
BB
EQUITY
High risk, high yield

BASIC SETUP

- N defaultable obligors (names): O_i
  where $i \in \mathcal{N} = \{1, 2, \cdots, N\}$
- stopping time ν_i: default time of O_i
- $1 - R_i$: proportional nominal loss of O_i, where $R_i \in (0, 1)$
- loss process $L = (L_t)_{t \geq 0}$

$$\boxed{L_t := \sum_{i=1}^{N} (1 - R_i) \mathbb{1}_{\{\nu_i \leq t\}}}$$

A credit derivative is a c.c. with payoff depending on L.

- exogenous process Y: macroeconomic factors
  impact of Y on credit default probabilities; see Bonfim (2009) and Chen (2010)

LITERATURE ON DEFAULT MODELS

1. Structure models
   Merton (1974), Black and Cox (1976), ...
   asset value $<$ debt (barrier) $\Rightarrow$ default
2. Copula models
   Li (2000), Hull and White (2004), ...
   copula to model joint distribution of defaults
3. Intensity-based models

LITERATURE ON INTENSITY-BASED MODELS

Two dominating approaches:

1. bottom-up approach
   Jarrow and Turnbull (1995), Lando (1998), Duffie and Singleton (1999), Giesecke and Weber (2006), Cvitanić et al. (2012) ...
   specify the intensity process λ_i for each name O_i s.t.

$$\left(\mathbb{1}_{\{\nu_i \leq t\}} - \int_0^t \lambda_i(s)\mathrm{d}s\right)_{t\geq 0} \quad \text{is a martingale}$$

2. top-down approach
   Errais et al. (2007, 2010), Giesecke et al. (2011), Cont and Minca (2013), ...
   specify the intensity process λ_L for the aggregate loss L s.t.

$$\left(L_t - \int_0^t \lambda_L(s)\mathrm{d}s\right)_{t\geq 0} \quad \text{is a martingale}$$

Outline

Definition 1

$X = (X_t)_{t\geq 0}$ denotes the default process, where X_t is the set of names that have defaulted by time t.
X is a set-value process taking values in subsets of $\mathcal{N}$.
$|X_t|$: number of elements in set X_t (cardinality).

Example. O_1 defaults at $t = 0.1$, O_5 at $t = 0.5$, and O_9 at $t = 1$; then $X_1 = \{1, 5, 9\}$ and $|X_1| = 3$.

Standing Assumptions

(i) No more than one default occurs at the same time
(ii) Obligors do not recover after the default

- $\mathbb{P}(|X_{t+\Delta t}| - |X_t| > 1) = o(\Delta t)$ and $\tau_1 < \cdots < \tau_i < \cdots < \tau_N$
- X is a non-decreasing process, i.e., $X_s \subseteq X_t$ for all $0 \leq s < t$
- The process $|X|$ jumps up by size 1 at default time τ_i, $i \in \mathcal{N}$

Filtrations

- $\mathbb{F}^Y = (\mathcal{F}_t^Y)_{t\geq 0}$: generated by the macroeconomic process Y
- $\mathbb{F}^X = (\mathcal{F}_t^X)_{t\geq 0}$: generated by the default process X
- $\mathbb{N}$: all the subsets of $\mathcal{N}$, i.e., $\mathbb{N} = 2^{\mathcal{N}}$ ($\mathcal{N} = \{1, 2, \cdots, N\}$)

Definition 2

A continuous time $\mathbb{N}$-valued stochastic process $X = (X_t)_{t\geq 0}$ is called an $\mathbb{F}^Y$-conditional Markov chain if, for all $0 \leq s \leq t$ and $F \in \mathbb{N}$, the following condition holds:

$$\mathbb{P}\left(X_t = F \mid \mathcal{F}_s^X \vee \mathcal{F}_s^Y\right) = \mathbb{P}\left(X_t = F \mid \sigma(X_s) \vee \mathcal{F}_s^Y\right), \quad \mathbb{P}\text{-a.s.}$$

Conditioning on Y, (the default process) X is a Markov chain.

Definition 3

A family of $\mathbb{F}^Y$-adapted processes $\boldsymbol{\Lambda} = (\Lambda_{EF}(t))_{t\geq 0}$ or $\boldsymbol{\lambda} = (\lambda_{EF}(t))_{t\geq 0}$, where $E \subseteq F \in \mathbb{N}$, is called the default intensity family of an $\mathbb{N}$-valued process $X = (X_t)_{t\geq 0}$, if the process $X_F = (X_F(t))_{t\geq 0}$, defined by

$$X_F(t) := \mathbb{1}_F(X_t) - \sum_{E\subseteq F} \int_0^t \mathbb{1}_{\{X_s=E\}} \mathrm{d}\Lambda_{EF}(s)$$

is a martingale for all $F \in \mathbb{N}$ with respect to the filtration $(\mathcal{F}_t^X \vee \mathcal{F}_t^Y)_{t\geq 0}$, where

$$\Lambda_{EF}(t) := \int_0^t \lambda_{EF}(s)\mathrm{d}s$$

Think: $X_s = E \rightarrow X_t = F$, where $0 \leq s \leq t$ and $E \subseteq F$

Remarks

- The intensity family $\boldsymbol{\Lambda}$ or $\boldsymbol{\lambda}$ in Definition 3 plays an important role in the compensator of the default process X.
- $\lambda_{EF}(t)$ represents the conditional default rate at time t when obligors in set E have already defaulted.
- We have $\lambda_{EF}(t) = 0$, whenever $F \neq E \cup \{i\}, i \in E^c$.

... wait a second. So far the framework really looks like the Markov chain models, see Bielecki et al. (2011)

Key Difference

We show that for suitable $\boldsymbol{\Lambda}$ (or $\boldsymbol{\lambda}$) and Y, there exists an $\mathbb{F}^Y$-conditional set-valued Markov chain X taking values in $\mathbb{N}$. In comparison, the existing works usually begin with such a Markov chain.
Benefit: apply the market prices/spreads to recover default intensities

Outline

Existence assumptions

Assumption 4

$\boldsymbol{M} = (M_{EF}(t))_{t\geq 0}$ *is a family of Poisson processes with intensity 1, where* $E \subseteq F$. $\boldsymbol{M}$ *and* Y *are independent.*

Assumption 5

The intensity family $\boldsymbol{\lambda} = (\lambda_{EF}(t))_{t\geq 0}$ *satisfies*

- $\lambda_{EF}(t) = 0$, *if* $E \neq F$ *or* $F \neq E \cup \{i\}$, *where* $i \in E^c$.
- $\lambda_E(t) := -\lambda_{EE}(t) = \sum_{E\neq F} \lambda_{EF}(t)$.
- $\lambda_{EF}(t) \geq 0$ *for all* $F = E \cup \{i\}$, *where* $i \in E^c$.
- $\lim\limits_{t\to+\infty} \int_0^t \lambda_{EF}(s)\mathrm{d}s = +\infty$ *for all* $F = E \cup \{i\}$, *where* $i \in E^c$.

Note. We can formulate Assumption 5 using $\boldsymbol{\Lambda}$.

Existence result

Theorem 6

Let Assumptions 4 and 5 hold, there exists an $\mathbb{F}^{Y}$-conditional Markov chain X with intensity family $\boldsymbol{\lambda}$ and $X_0 = \emptyset$.

Model Flexibility:

- Y is arbitrary
- Minimum assumptions on the default intensity family $\boldsymbol{\lambda}$

Dynamics of X

$F \backslash E = \{x : x \in F \text{ and } x \notin E\}$, where $E \subset F$ and $|F \backslash E| = n$

$\Pi(F \backslash E)$ denotes the set of all the permutations of $F \backslash E$

$\forall \boldsymbol{\pi} \in \Pi(F \backslash E)$, define a sequence of sets $(F_k^{\boldsymbol{\pi}})_{k=0,1,\cdots,n}$ by

$$F_0^{\boldsymbol{\pi}} := E \quad \text{and} \quad F_k^{\boldsymbol{\pi}} := F_{k-1}^{\boldsymbol{\pi}} \cup \{\pi_k\}, \ \ k = 1, 2, \cdots, n$$

Remark. $E \to F_1^{\boldsymbol{\pi}} \to F_2^{\boldsymbol{\pi}} \to \cdots \to F_n^{\boldsymbol{\pi}} = F$ is a default path

Example. Let $E = \{1, 2\}$ and $F = \{1, 2, 3, 4\}$. Suppose $X_s = E$ and $X_t = F$, where $s < t$. Since $\Pi(F \backslash E) = \{(3, 4), \ (4, 3)\}$, from s to t, obligors O_3 and O_4 have defaulted, and the path is
$F_0^{\boldsymbol{\pi}_1} = \{1, 2\} \to F_1^{\boldsymbol{\pi}_1} = \{1, 2, 3\} \to F_2^{\boldsymbol{\pi}_1} = \{1, 2, 3, 4\}$
or $F_0^{\boldsymbol{\pi}_2} = \{1, 2\} \to F_1^{\boldsymbol{\pi}_2} = \{1, 2, 4\} \to F_2^{\boldsymbol{\pi}_2} = \{1, 2, 3, 4\}$

Theorem 7

Let Assumptions 4 and 5 hold. $\forall\, s \le t$ *and* $F \in \mathbb{N}$*, we have, for any bounded* $\mathcal{F}_t^Y$*-measurable random variable* ξ*, that*

$$\mathbb{E}\left[\mathbb{1}_{\{X_t=F\}} \cdot \xi \mid \mathcal{F}_s^X \vee \mathcal{F}_s^Y\right] = \sum_{E \subseteq F} \mathbb{1}_{\{X_s=E\}} \cdot \mathbb{E}\left[\xi\, G(s,t;E,F) \middle| \mathcal{F}_s^Y\right]$$

$$H_0(s,t;E) := e^{-\int_s^t \lambda_E(u)\mathrm{d}u}, \text{ with } \lambda_E(t) = -\lambda_{EE}(t) = \textstyle\sum_{E \neq F} \lambda_{EF}(t)$$

$$H_{k+1}(s,t;\cdots) := \int_s^t \lambda_{F_k^{\boldsymbol{\pi}} F_{k+1}^{\boldsymbol{\pi}}}(v) \cdot e^{-\int_v^t \lambda_{F_{k+1}^{\boldsymbol{\pi}}}(u)\mathrm{d}u} \cdot H_k(s,v;\cdots)\mathrm{d}v$$

$$G(s,t;E,F) := \begin{cases} H_0(s,t;E), & \text{if } E = F \\ \displaystyle\sum_{\boldsymbol{\pi} \in \Pi(F \setminus E)} H_{|F\setminus E|}(s,t;F_0^{\boldsymbol{\pi}},\cdots,F_{|F\setminus E|}^{\boldsymbol{\pi}}), & \text{if } E \subset F \end{cases}$$

REMARKS

Given $X_t = F$, $X_s = E = F$ or $E \subset F$, explaining $\sum_{E \subseteq F} \mathbb{1}_{\{X_s = E\}}$

$H_k(s, t; F_0^{\pi}, F_1^{\pi}, \cdots, F_k^{\pi})$: probability that X evolves from $X_s = E = F_0^{\pi}$ to $X_t = F_k^{\pi}$ (with k defaults) in a particular path

Hence, $G(s, t; E, F)$ captures exactly the transition probability of X from $X_s = E$ to $X_s = F$

Remark. If $N = 1$ or 2, we can obtain very simplified results on $\mathbb{P}\left[X_t = F \mid \mathcal{F}_s^X \vee \mathcal{F}_s^Y\right]$
$\Rightarrow$ potential applications to the FTD (first-to-default baskets), where $N = 5$

If N is large (e.g., $N = 125$ for iTraxx), the computations are intensity due to the involvement of permutations.

INTENSITY MODELING

Assumption 8

Let constants $\beta_i, \rho_{ji} > 0$ *and function* $h(\cdot)$ *be positive with* $h(0) = 1$. *We define, for all* $E \in \mathbb{N}$ *and* $i \in E^c$, *that*

$$\mathcal{L}_E(i) := \begin{cases} h(|E|) \cdot \sum_{j \in E} \rho_{ji}, & \text{if } E \neq \emptyset \\ \beta_i, & \text{if } E = \emptyset \end{cases}$$

$$\text{and} \quad \overline{\mathcal{L}}_E := \sum_{i \in E^c} \mathcal{L}_E(i), \quad \text{with} \quad \overline{\mathcal{L}}_{\mathcal{N}} := 0$$

Let $\Phi(\cdot, \cdot)$ *be a positive functional mapping from* $[0, \infty) \times \mathbb{R}^d$ *to* $\mathbb{R}^+$. *The intensity family* $\boldsymbol{\lambda} = (\lambda_{EF}(t))_{t \geq 0}$ *is given by*

$$\lambda_{EF}(t) = \begin{cases} \Phi(t, Y_t) \cdot \mathcal{L}_E(i), & \text{if } F = E \cup \{i\} \text{ and } i \in E^c \\ -\Phi(t, Y_t) \cdot \overline{\mathcal{L}}_E, & \text{if } E = F \\ 0, & \text{otherwise} \end{cases}$$

Model explanations

- β_i: base default intensity of O_i (no contagion)
- ρ_{ji}: individual contagion rate of O_j on O_i
  Recall $j \in E$ (defaulted set) and $i \in E^c$ (surviving set)
- $h : \{0, 1, \cdots, N\} \to \mathbb{R}^+$: impact of default magnitude
- $\mathcal{L}_E(i)$: intergroup contagion effect of E on obligor O_i
- $\overline{\mathcal{L}}_E$: aggregate impact of defaulted obligors in E on all survivors in E^c
- Φ: contagion effect of macroeconomic factors

Note. With Assumption 8 on the intensity family $\boldsymbol{\lambda}$, we can further reduce the results of Theorem 7 (conditional probability and expectation of X).

AFFINE JUMP-DIFFUSION Y

Assumption 9

(i) $\Phi(t, y) = y$ *for all* $t \geq 0$.
(ii) The macroeconomic factor process Y *is given by*

$$dY_t = \kappa(\theta - Y_t)dt + \sigma\sqrt{Y_t}dW_t + dJ_t, \qquad \textit{with} \quad Y_0 = y_0$$

$\kappa, \theta, \sigma > 0$, *W B.M.*, *J compound Poisson (primary parameter l and secondary exponential with mean* μ*). W and J are independent.*

Duffie and Garleanu (2001) and Mortensen (2006):

$$\mathbb{E}\left[e^{-g\int_0^t Y_s ds}\right] = e^{A(g,0,t)+y_0 \cdot B(g,0,t)}, \qquad g > 0$$

CDO

Consider the proportion version of the loss process

$$L_t = \frac{R_X(t)}{N} = \frac{\sum_{i \in X_t}(1 - R_i)}{N}, \qquad t \geq 0$$

Attach points are $0 < \cdots < p_K \leq 1$ and tranche i is $[p_{i-1}, p_i]$.
The accumulated loss of tranche i is defined by

$$\boxed{L^{(i)}(X_t) := (L_t - p_{i-1})^+ - (L_t - p_i)^+}$$

- (Default Leg) The protection seller covers $L^{(i)}(X_t)$.
- (Premium Leg) The protection buyer pays upfront fees $u^{(i)}\Delta p_i = u^{(i)} \times (p_i - p_{i-1})$ at inception and periodic premiums or spreads $s^{(i)}(\Delta p_i - L^{(i)}(X_{t_{k-1}}))\Delta_k$ at each payment time t_k, where $k = 1, \cdots, m$.
($\Delta_k = 1/4$ quarterly payments and m is the term.)

Spreads

Proposition 10

$$s^{(i)} = \frac{\sum_{k=1}^{m} e^{-rt_k}\mathbb{E}\left[L^{(i)}(X_{t_k}) - L^{(i)}(X_{t_{k-1}})\right] - u^{(i)}\Delta p_i}{\sum_{k=1}^{m} e^{-rt_k}\left(\Delta p_i - \mathbb{E}[L^{(i)}(X_{t_{k-1}})]\right)\Delta_k}$$

where $\mathbb{E}$ denotes expectation under the risk neutral probability.

$$\mathbb{E}[L^{(i)}(X_{t_k})] = \sum_{n=0}^{N} \sum_{F\in\mathcal{A}(n)} \sum_{\boldsymbol{\pi}\in\Pi(F\setminus\emptyset)} \sum_{j=0}^{n} L^{(i)}(F)\widehat{\mathcal{L}}^{\boldsymbol{\pi}}(n)\alpha_j^{(n)}(\boldsymbol{\pi}) \cdot \mathbb{E}\left[e^{-\overline{\mathcal{L}}_{F_j^{\boldsymbol{\pi}}}\cdot\int_0^{t_k}\Phi(u,Y_u)\mathrm{d}u}\right]$$

EXAMPLE I

Homogeneous Contagion Model (HCM)

Let (intensity) Assumption 8 hold. We assume $\rho_{ij} = \rho$ for all $i \neq j$, $\Phi(t, y) = y$ and $h(n) = e^{-\delta n}$, where δ is a constant.

Proposition 11

Let Assumptions 4 and 9 hold. Under the HCM, we have

$$\mathbb{E}\left[L^{(i)}(X_{t_k})\right] = \sum_{j=0}^{N-1} \Gamma_j^{(i)} \cdot \exp\left(A(a_j, 0, t_k) + y_0 B(a_j, 0, t_k)\right) + 1$$

We can also compute $\mathbb{P}(|X_t| = n)$ explicitly.

Note. Γ_j and a_j are explicitly given.

Outline

PART I. TOY EXAMPLES

- Number of obligors: $N = 125$
- Risk-free interest rate: $r = 5\%$
- Payment frequency: $\Delta = 1/4$ (quarterly)
- Recovery rate: $R_i \equiv R = 40\%$
- Process Y (taken from Duffie and Garleanu (2001)):
  $y_0 = 1,\ \kappa = 0.6,\ \theta = 0.02,\ \sigma = 0.141,\ l = 0.2,\ \text{and}\ \mu = 0.1$
- HCM parameters:
  $\rho = 0.05$, $\delta = -0.008$, and $a_0 = 0.35$

Table 1: 5-year CDO Tranche Spreads under HCM and NCM

Tranches	HCM Spread (bp)	NCM Spread (bp)
$[0, 3\%]$	1502	918
$[3\%, 6\%]$	1240	590
$[6\%, 9\%]$	1095	511
$[9\%, 12\%]$	977	435
$[12\%, 22\%]$	839	359
$[22\%, 60\%]$	619	283

Note. NCM stands for Near-neighbor Contagion Model, where each obligor O_i only impacts its two neighbors O_{i-1} and O_{i+1}.

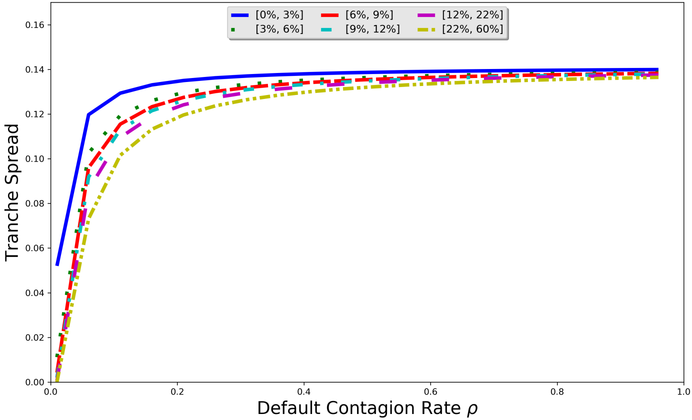
Table 2: Attachment and Detachment Time under HCM

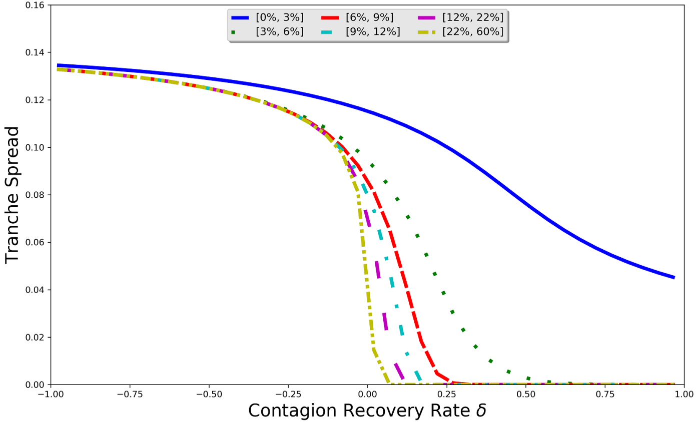
Tranches	Detachment #	Attachment t	Detachment t
$[0, 3\%]$	7	0.5	1.25
$[3\%, 6\%]$	13	1.25	1.75
$[6\%, 9\%]$	19	1.75	2.25
$[9\%, 12\%]$	25	2.25	3
$[12\%, 22\%]$	46	3	11
$[22\%, 60\%]$	125	11	294

Note. t is in unit of years.

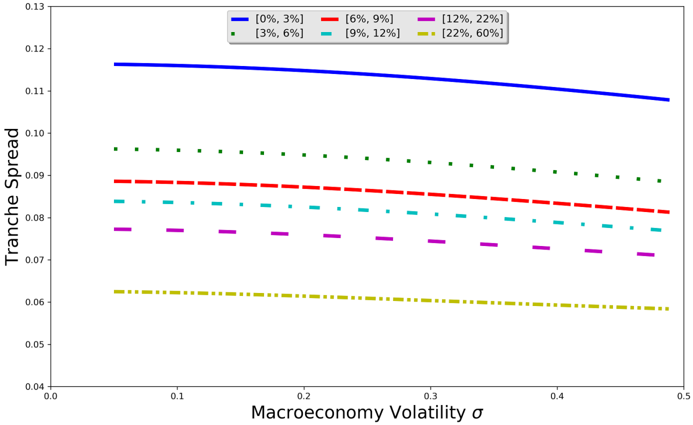
$3\% \times N(125)/60\% = 6.25 \Rightarrow [0, 3\%]$ detach at the 7th default

Figure 1: Sensitivity Analysis of CDO Tranche Spreads under HCM





Recall $h(n) = e^{-\delta n}$ measures the magnitude


[0%, 3%]
[3%, 6%]
[6%, 9%]
[9%, 12%]
[12%, 22%]
[22%, 60%]
Tranche Spread
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.0
0.1
0.2
0.3
0.4
0.5
Macroeconomy Volatility σ

In addition, we have also considered recovery rate R, number of payments m, mean-reversion speak κ of Y.

- The CDO tranche spreads are very sensitive to all factors considered here, except for the macroeconomy volatility σ.
- Among all six factors considered, only the default contagion rate ρ is positively related with respect to the tranche spreads, while the rest shows negative relation.
- The tranche spreads are extremely elastic to the default contagion rate ρ and contagion recovery rate δ. One can interpret δ as the government intervention or self recovery rate of the group. The equity tranche is less sensitive to δ comparing with other tranches since it mainly reflects idiosyncratic risk.

Part II. Market Calibration

- Data: 5-year CDX North American Investment Grade (5Y CDX.NA.IG) from Seo and Wachter (2018)
- Attachment points: 0, 3, 7, 10, 15, 30 (in percentage)
- Full sample: 10/05 - 9/08; Pre-crisis sample: 10/05 - 9/07; Post-crisis sample: 10/7 - 9/08
- Goal: estimate $\boldsymbol{x} = (a_0, \kappa, \theta, \sigma, \mu, l, \delta, \rho, y_0)$
- Best fit $\hat{\boldsymbol{x}}$:

$$\min_{\boldsymbol{x}} \sum_{i=1}^{6} \left(\frac{Tranche\ i^{model} - \overline{Tranche\ i^{market}}}{\overline{Tranche\ i^{market}}} \right)^2$$

Table 3: Calibrated Parameters

Parameters	Full	Pre-crisis	Post-crisis
a_0	1.9762	1.1985	1.8978
κ	0.5626	0.5631	0.3619
θ	0.4428	0.1765	0.6893
σ	0.1197	0.0743	0.1984
μ	2.5805	1.8237	3.0000
l	0.5138	2.0000	0.2537
δ	-0.0098	-0.0269	0.0079
ρ	0.0025	0.0014	0.0039
y_0	1.8460	0.9974	2.1535

Note. $a_0 = \sum_{j=1}^{N} \beta_i$ (aggregate base default rates).
$h(n) = e^{-\delta n}$, $\delta < 0$ (resp. $\delta < 0$) implies positive (negative) effect on credit spreads.
ρ (default intensity) almost tripled from 0.0014 to 0.0039.

Table 4: Calibration of 5Y CDX.NA.IG Tranches and Index

	Full		Pre-crisis		Post-crisis	
	Data	Model	Data	Model	Data	Model
$[0, 3\%]$	39	26	31	17	54	37
$[3\%, 7\%]$	238	222	108	88	498	498
$[7\%, 10\%]$	102	96	25	26	255	229
$[10\%, 15\%]$	54	56	12	13	136	145
$[15\%, 30\%]$	27	26	6	6	69	65
$[30\%, 100\%]$	NA	11	NA	1	NA	27
Index	67	87	42	55	116	142

Table 5: Implied Default Contagion Rate ρ

	Full	Pre-crisis	Post-crisis
$[0, 3\%]$	0.116%	0.045%	0.177%
$[3\%, 7\%]$	0.086%	0.035%	0.136%
$[7\%, 10\%]$	0.083%	0.033%	0.129%
$[10\%, 15\%]$	0.081%	0.033%	0.112%
$[15\%, 30\%]$	0.030%	0.035%	0.062%

implied ρ: model = data

implied default contagion rate smile

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Thank you!