Personalized Robo-Advising:
Enhancing Investment through Client Interaction

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Robo-Advisors

Automated investment platforms providing algorithm-driven investment advice with limited human supervision

- First robo-advisors launched in the wake of the 2008 financial crisis (Betterment, Wealthfront, Personal Capital, . . .)
- Current offerings include:
  - Affordable portfolio management (fees, account size, etc.)
  - Full Automation (portfolio construction and rebalancing)
  - Tax-loss harvesting
- Becoming both day-to-day and long-term money managers
Robo-Advisors

- Currently manage around $750 billion in the United States (2019)
- Less than 1.5% of total investable assets

Assets under Management in the Robo-Advisors market

Source: Statista, September 2019
Existing Literature

- D’Acunto et al. [2018] and Rossi and Utkus [2019a,b]
  - Robo-advising vs. traditional financial advising
- Reher and Sokolinski [2020]
  - Robo-advising and financial inclusion
- Beketov et al. [2018] analyze over 200 robo-advisors globally
  - Risk profiling based on online questionnaires
  - Mean-variance portfolio optimization
- Classification of robo-advising systems (D’Acunto and Rossi [2020])
  - Portfolio personalization
  - Client involvement
  - Level of human-advising
Model of Robo-Advising

We propose the first quantitative model of robo-advising:

- Investment process accounts for **dynamic** risk preferences and **repeated** interaction between client and robo-advisor

Our framework is consistent with that of the most prominent stand-alone robo-advising firms:

- High portfolio personalization
- Low and indirect client involvement
- Limited or no human-advising
Client:
- Dynamic risk aversion process: \((\gamma_n^C)_{n\geq 0}\)
- Provides information to the robo-advisor at interaction times

Robo-Advisor:
- Constructs a *model* of the client’s risk aversion: \((\gamma_n^R)_{n\geq 0}\)
- Differs from \((\gamma_n^C)_{n\geq 0}\) due to imperfect human-machine interaction
  - Changes to client’s demographics only observed at interaction times
  - Information communicated by client affected by **behavioral** biases
- Designs an optimal investment strategy
Determinants of Client’s Risk Preferences

Client’s risk aversion \((\gamma_n^C)_{n \geq 0}\) changes because of:

- Passage of time
  - 1st generation robo-advisors \(\approx\) TDFs
- Shocks to demographics
  - Barsky et al. [1997], Guiso and Paiella [2008], ...
- Market returns and economic conditions
  - Fama and French [1989], Cohn et al. [2015], Bucciol and Miniaci [2018], Guiso et al. [2018]
Research Questions

(1) Does stochastic variation in the client’s risk aversion produce an intertemporal hedging demand for the risky asset?

(2) Does frequent interaction allow the robo-advisor to implement a strategy closely matching the client’s risk profile?

(3) Should the robo-advisor always cater to the client’s wishes?
Model components:

- Market model for available investment securities
- **Dynamic** model for client’s risk aversion
- Mechanism to decide the human-machine interaction schedule
- Optimal adaptive investment criterion
Market Dynamics

- Risky asset \((S_n)_{n \geq 0}\) and a risk-free asset \((B_n)_{n \geq 0}\),

\[
S_{n+1} = (1 + Z_{n+1}(Y_n))S_n
\]
\[
B_{n+1} = (1 + r(Y_n))B_n
\]

- Price dynamics modulated by a Markov regime switching model of economic conditions \((Y_n)_{n \geq 0}\) (Hamilton [1989])

- Given \(Y_n = y\), the risk-free rate is \(r(y) \geq 0\), and the risky asset’s return has mean \(\mu(y) > r(y)\), and variance \(\sigma^2(y) > 0\)

- Probability space \((\Omega, \mathcal{F}, \mathbb{P})\) also supports a sequence \((\epsilon_n)_{n \geq 1}\) of random variables, independent of \((Y_n)_{n \geq 0}\) and \((Z_n)_{n \geq 1}\)

- Filtration \((\mathcal{F}_n)_{n \geq 0}\) defined by \(\mathcal{F}_n = \sigma(Y_{(n)}, Z_{(n)}, \epsilon_{(n)})\)
Self-financing investment strategy $\pi = (\pi_n)_{n \geq 0}$

- $\pi_n$ is the amount of wealth invested in the risky asset at time $n$

Under $\pi$, the client’s wealth process $(X_n^\pi)_{n \geq 0}$ satisfies

$$X_{n+1}^\pi = R_{n+1}X_t^\pi + \tilde{Z}_{n+1}\pi_n$$

where $R_{n+1} := 1 + r(Y_n)$ and $\tilde{Z}_{n+1} := Z_{n+1}(Y_n) - r(Y_n)$
Client’s Risk Aversion Process

- Captures stylized features of retail investors’ risk profiles:

\[ \gamma_n^C := \gamma_n^C(Y(n), Z(n), \epsilon(n)) = e^{\eta_n} \gamma_n^{id} \gamma_n^Y \]

- The first component \((e^{\eta_n}) \geq 0\) is of the form

\[ e^{\eta_n} = e^{-\alpha(T-n)} \]

and captures age-related increase in risk aversion

- The second component \((\gamma_n^{id}) \geq 0\) is of the form

\[ \gamma_n^{id} = \gamma_{n-1}^{id} e^{\epsilon_n}, \quad \epsilon_n = \begin{cases} \mathcal{N}(0, \sigma_\epsilon^2), & \text{w.p. } p_\epsilon \\ 0, & \text{w.p. } 1 - p_\epsilon \end{cases} \]

and captures idiosyncratic shocks to the client’s risk aversion
The third component is a state-dependent coefficient $\gamma_n^Y = \bar{\gamma}(Y_n)$, which is increasing in the market Sharpe ratio:

$$\frac{\mu(y_i) - r(y_i)}{\sigma(y_i)} \geq \frac{\mu(y_j) - r(y_j)}{\sigma(y_j)} \implies \tilde{\gamma}(y_i) \geq \tilde{\gamma}(y_j)$$

Risk aversion and market Sharpe ratio ($\lambda$) are countercyclical

- Higher at business cycle troughs than at peaks
- Lettau and Ludvigson [2010], Campbell and Cochrane [1999]:

$$Y \downarrow \implies \gamma^C \uparrow \implies \lambda \uparrow \implies S \downarrow$$

“Unadvised” client is inclined to reduce market exposure when the market Sharpe ratio is high
The interaction schedule \( (T_k)_{k \geq 0} \) is an increasing sequence of stopping times with respect to the filtration \((F_n)_{n \geq 0}\).

Interaction can be triggered by any combination of client-specific events, economic state changes, and market events.

Define \((\tau_n)_{n \geq 0}\) where \(\tau_n := \sup\{T_k : T_k \leq n\}\) is the most recent interaction time occurring prior to or at time \(n\).

**Deterministic schedule:** The sequence \((T_k)_{k \geq 0}\) is given by

\[
T_k = k\phi, \quad k \geq 0
\]

where \(\phi \geq 1\) is the time between consecutive interactions.
At an interaction time $n$, the risk aversion value communicated by the client is

$$\xi_n = \gamma_n^C \gamma_n^Z$$

where $\gamma_n^C$ is the client’s risk aversion and, for $\beta \geq 0$,

$$\gamma_n^Z = e^{-\beta \left( \frac{1}{\phi} \sum_{k=n-\phi}^{n-1} (Z_{k+1} - \mu_{k+1}) \right)}$$

captures client’s behavioral biases (e.g., trend-chasing)

The bias $\gamma_n^Z$ is based on recent stock market performance:
- Market outperforming the client’s expectations: $\gamma_n^Z < 1$
- Market underperforming the client’s expectations: $\gamma_n^Z > 1$
Magnitude of the bias $\gamma_n^Z$ is increasing in the sensitivity $\beta$ but decreasing in the time between interactions $\phi$:

$$\mathbb{E}_n[\gamma_n^Z] \approx e^{\frac{1}{2} \frac{\beta^2 \sigma^2(\gamma_n)}{\phi}}$$

Sensitivity $\beta$ assumed known to the robo-advisor at the outset

- Behavioral biases related to financial literacy, experience, and cognitive abilities (e.g., Oechssler et al. [1997], Seasholes and Feng [2005])
Robo-Advisor’s Model of Client

- Robo-advisor’s filtration \((\mathcal{F}_n^R)_{n \geq 0}\) is generated by

\[
D_n := (Y(n), Z(n), \tau(n), \xi(n))
\]

which consists of market information \((Y(n), Z(n))\) and information from client-interaction \((\tau(n), \xi(n))\).

- Robo-advisor’s model of the client’s risk aversion \((\gamma_n^R)_{n \geq 0}\) is a process adapted to the robo-advisor filtration:

\[
\gamma_n^R := \gamma_n^R(D_n)
\]

- The filtration \((\mathcal{F}_n^R)_{n \geq 0}\) grows with the frequency of interaction.
The robo-advisor’s model \( (\gamma^R_n)_{n \geq 0} \) is given by

\[
\gamma^R_n = \mathbb{E}_n[\gamma^C_n] \gamma^Z_{\tau_n} = \xi_{\tau_n} e^{\eta_n - \eta_{\tau_n}} \frac{\gamma^Y_n}{\gamma^Y_{\tau_n}}
\]

where \( \gamma^C_n \) is the client’s risk aversion and \( \gamma^Z_{\tau_n} \) is the bias realized at the previous time of interaction.

Updated in real time based on the passage of time, realized market returns, and changes in economic conditions.
Adaptive Mean-Variance Criterion

• **Partial Equilibrium framework:** For a fixed horizon $T \geq 1$, the robo-advisor maximizes a **mean-variance** objective

$$J_n(x, d, \pi) := \mathbb{E}_{n, x, d} \left[ \frac{X_T^{\pi} - X_n}{X_n} \right] - \frac{\gamma_n^R}{2} \text{Var}_{n, x, d} \left[ \frac{X_T^{\pi} - X_n}{X_n} \right]$$

where $\pi$ is a self-financing strategy

• Initial condition fixes the robo-advisor’s information set:
  
  $$\mathbb{P}_{n, x, d}(\cdot) := \mathbb{P}(\cdot | X_n = x, D_n = d)$$

• Sequence of objective functions $(J_n)_{0 \leq n < T}$
  
  • Robo-advisor’s model of client’s risk preferences $\gamma_n^R > 0$ adapts to market, economic, and client-communicated information

• **Time-inconsistent stochastic control problem**
Multi-period mean-variance optimization is time-inconsistent

- Tower property of conditional expectations does not apply to $(\mathbb{E}_{n,x,d}[X_T])^2 \implies \text{Bellman optimality principle does not hold}$
- Stochastic risk-return coefficient is an additional source of time-inconsistency

- Multitude of future decision-making “selves” that may not act in the best interest of previous “selves”
  - If $\pi^*$ is the control law that maximizes $J_n$, then $\pi^*$ restricted to \{\(n + 1, n + 2, \ldots, T\}\} is suboptimal for the objective function $J_{n+1}$
Adaptive Mean-Variance Criterion

- **Myopic investment**
  - At each time $n$, apply the control $\pi_n^*$ where $\pi^*$ is the control law that maximizes $J_n$ (reoptimize at each time)
  - Ignores time-inconsistency

- **Pre-committed investment**
  - At time $n = 0$, find the control law $\pi^*$ that maximizes $J_0$ and commit to this strategy throughout the investment horizon
  - Ties the hands of future “selves”

- **Equilibrium control**
  - At each time $n$, apply the control $\pi_n^*$ that maximizes $J_n$, given that future “selves” will act in their own best interest
  - Non-cooperative game with one player for each time $n$
  - Subgame-perfect equilibrium
Look for an optimal control $\pi^*$ which is time-consistent:

$$\sup_{\pi \in A^*_{n+1}} J_n(\pi) = J_n(\pi^*), \text{ for all } 0 \leq n < T$$

where $A^*_{n+1} = \{\pi : \pi_{n+1:T} = \pi^*_{n+1:T}\}$ is the set of control laws that coincide with $\pi^*$ after time $n$

Any candidate optimal control $(\pi_n)_{n \geq 0}$ is of the form

$$\pi_n = \pi_n(x, d) \in \mathcal{F}_n^R$$

such that $\mathbb{E}_0\left[\sum_{n=0}^{T-1} \pi_n^2\right] < \infty$
The optimal proportion of wealth invested in the risky asset at time $n$ is

$$\pi_n^* = \frac{1}{\gamma_n^R} \frac{\mathbb{E}_n[\tilde{Z}_{n+1} r_{n+1}^{\pi^*}]}{\text{Var}_n[\tilde{Z}_{n+1} r_{n+1}^{\pi^*}]} - R_{n+1} \frac{\text{Cov}_n(r_{n+1}^{\pi^*}, Z_{n+1} r_{n+1}^{\pi^*})}{\text{Var}_n[\tilde{Z}_{n+1} r_{n+1}^{\pi^*}]}$$

where $\tilde{Z}_{n+1}$ is the excess market return at $n+1$, and

$$r_{n+1}^{\pi^*} = \frac{X_{T}^{\pi^*}}{X_{n+1}}$$

is the value of one dollar invested in the optimal portfolio between time $n+1$ and the terminal date $T$.

The allocation $\pi_n^*$ depends on both the current risk-return tradeoff $\gamma_n^R$ and its future dynamics through the conditional expectations.
Structure of Optimal Strategy

\[ \pi_n^* = \frac{1}{\gamma_n R_n} \frac{\mathbb{E}_n[\tilde{Z}_{n+1} r_{n+1}^{\pi^*}]}{\text{Var}_n[\tilde{Z}_{n+1} r_{n+1}^{\pi^*}]} - R_{n+1} \frac{\text{Cov}_n(r_{n+1}^{\pi^*}, Z_{n+1} r_{n+1}^{\pi^*})}{\text{Var}_n[\tilde{Z}_{n+1} r_{n+1}^{\pi^*}]} \]

- \( \frac{1}{\gamma_n R_n} \frac{\mathbb{E}_n[Z_{n+1} r_{n+1}^{\pi^*}]}{\text{Var}_n[Z_{n+1} r_{n+1}^{\pi^*}]} \): standard single-period Markowitz strategy, but also accounting for the future portfolio return \( r_{n+1}^{\pi^*} \)

- \( \frac{\text{Cov}_n(r_{n+1}^{\pi^*}, Z_{n+1} r_{n+1}^{\pi^*})}{\text{Var}_n[Z_{n+1} r_{n+1}^{\pi^*}]} \): intertemporal hedging component

Incorporates the effect of market returns and economic conditions on the client’s risk aversion

\[ \tilde{Z}_{n+1} \uparrow \implies \gamma_{n+1} \downarrow \implies \pi_{n+1}^* \uparrow \implies r_{n+1}^{\pi^*} \uparrow \implies \text{Cov}_n(\ldots) > 0 \]
Portfolio Personalization

- Relative difference between the robo-advisor’s model and the client’s risk aversion process:

\[
\mathcal{R}(\phi, \beta) := \mathbb{E} \left[ \frac{1}{T} \sum_{n=0}^{T-1} \left| \frac{\gamma_n^R - \gamma_n^C}{\gamma_n^C} \right| \right]
\]

- \( \phi \geq 1 \): time between consecutive interaction times
- \( \beta \geq 0 \): strength of client’s behavioral bias

- Robo-advisor faces a tradeoff between information acquisition rate and accuracy of acquired information

**Proposition**

1. There exists a unique value of \( \phi \) that minimizes \( \mathcal{R}(\phi, \beta) \)
2. Optimal value of \( \phi \) is increasing in \( \beta \)
Optimal Interaction Frequency

- Magnitude of behavioral bias increases with the interaction frequency
  - Myopic loss aversion (Benartzi and Thaler [1995])
Economy with two states $\mathcal{Y} = \{1, 2\}$ corresponding to economic expansions and contractions, and such that

$$\frac{\mu(1) - r(1)}{\sigma(1)} < \frac{\mu(2) - r(2)}{\sigma(2)}$$

Consider an investment strategy of the form

$$\pi_n^*(y) = \begin{cases} \bar{\pi}, & y = 1 \\ \bar{\pi}(1 + \delta), & y = 2 \end{cases}$$

- $\delta < 0$: “unadvised” client shifting wealth away from risky asset when the market Sharpe ratio is high
- $\delta > 0$: “robo-advised” client doing the opposite
Proposition

The optimal portfolio’s Sharpe ratio $s^{\pi^*}(\delta)$ is increasing and concave around $\delta = 0$:

$$\frac{\partial s^{\pi^*}(\delta)}{\partial \delta} > 0, \quad \frac{\partial^2 s^{\pi^*}(\delta)}{\partial \delta^2} < 0$$

- Compared to a buy-and-hold strategy:
  - Sharpe ratio lower if client is left unassisted ($\delta < 0$)
  - Sharpe ratio higher with robo-advising ($\delta > 0$)
- The drop in Sharpe ratio is greater when tilting away from the risky asset compared to the gain when tilting towards the risky asset
Economic Transitions and Portfolio Returns

Left: Client \((\delta < 0)\) vs. Buy-and-Hold \((\delta = 0)\).
Right: Client \((\delta < 0)\) vs. Robo-Advisor \((\delta > 0)\).
Should the robo-advisor’s recommendations depend on the client’s account monitoring frequency?
- Effect of short-term losses outweighs the effect of short-term gains → gradual increase in risk aversion and shift to safer investments
- Gneezy and Potters [1997]: causal relationship between frequency at which returns are evaluated and willingness to accept risk
- Myopic loss aversion exacerbated by technology?

Threshold-based rebalancing (Beketov et al. [2018])
- Portfolio weights adjusted only if there are sufficiently large changes in market prices and client characteristics
Future Developments

- Strategically determine interaction times to avoid inflated risk aversion and ensure good investment performance
  - Take into account investment performance since the previous interaction time - reference point of client
  - Restrict interaction during bad economic conditions to keep client invested in stock market
- Strategically communicate investment performance to clients
  - Emphasize projected long-term wealth distribution
  - Display the performance of a well diversified portfolio rather than the performance of individual asset classes
Future Developments

- Robo-advisor’s recommendations accounting for uncertainty in elicitation of risk preferences
  - Noisy and biased risk aversion coefficients
  - Disentangling risk aversion from client expectations
  - Accuracy of elicitation methods depend on financial literacy and numerical skills (Dave et al. [2010])
- Clients may not be easily contacted or willing to respond
  - Investor attentiveness (e.g. Abel et al. [2007, 2013] and Gargano and Rossi [2018])
Thank you!


