Math 126 Final Exam

Wednesday, December 11th, 2024, 2:00–4:00pm

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam.

You must show all your work and carefully justify your methods to obtain full credit. Answers must be neat, organized and unambiguous. You must state and justify the methods used.

No cell-phone or calculator are allowed. A double-sided, hand-written cheat sheet is allowed.

Full name (please print): _____

Signature: _____

Student ID: _____

☐ G. Reyes 10 am ☐ G. Reyes 1 pm ☐ Yinan Shen 9 am

☐ Zhengye Zhou 11am ☐ Zhengye Zhou 12 pm

1 (25 pts)	6 (30 pts)
2 (25 pts)	7 (30 pts)
3 (25 pts)	8 (30 pts)
4 (20 pts)	9 (20 pts)
5 (30 pts)	

235 points total

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1. Evaluate the limits

$a)$ $\lim_{x\to 0^+} (1 + x\ln(x))^{\frac{1}{x}}$

$b)$ $\lim_{x\to 0} \dfrac{\sin x - x\cos x - \sin(x^3/3)}{x^5}$

(12 pt. + 13 pt.)

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2. Compute the indefinite integrals

$$a)\ \int (x^2 - x + 1)\exp(x)\,dx \qquad\qquad b)\ \int \sin^3 x\,dx$$

(12 pt. + 13 pt.)

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3. Determine whether the following integrals are convergent or divergent. Justify your answer.

$$a) \int_0^1 \frac{1+\exp(-x)}{x^3}\,dx \qquad\qquad b) \int_1^{+\infty} \frac{2x}{x^3+1}\,dx$$

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(12 pt. + 13 pt.)

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4. Consider the region bounded by $y = x^2 - x^3$, and $y = 0$, where $x \geq 0$. Find an integral formula for the volume of the solid obtained

a) if we rotate the region about x-axis;

b) if we rotate the region about y-axis.

(do **not** calculate the actual values).

(10 pt. + 10 pt.)

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5. We intend to lift a basket to the top of a building by means of a flexible rope. Initially, half of the rope is lying on the ground and the other half is hanging from the top. The basket is attached to the end of the rope. If the total length of the rope is $50\,m$, its linear density is $1\,kg/m$ and the mass of the basket is $2\,kg$, find the amount of work needed to lift the basket to the top of the building. (Disregard frictional forces and consider the basket a point mass).

(30 pt.)

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6. Using appropriate tests, determine whether the series below converge or diverge. Show all work, by naming the test, applying the test, and drawing the correct conclusion.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n+5^n}$ *b)* $\displaystyle\sum_{n=3}^{\infty} \frac{1}{\sqrt[3]{4n^3+4n+1}}$ *c)* $\displaystyle\sum_{n=1}^{\infty} (n^2+1)\sin\frac{1}{n^2}$

(10 pt. + 10 pt. + 10 pt.)

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7. Given the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+2)\cdot 2^n}(x-5)^n$$

a) Find its radius of convergence;

b) Find its interval of convergence (that is, the set of points where it is convergent);

c) Find an upper bound for the remainder $|R_5|$ at the only point where the series converges conditionally.

(15 pt. + 10 pt. +5 pt.)

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8. We intend to find the area of the region bounded by the graph of $f(x) = \dfrac{1}{1+x}$, the X-axis and the lines $x = 0$ and $x = 0.1$. To that end, we approximate f near $x = 0$ by a Taylor polynomial. What should the degree of the polynomial be at least to achieve an accuracy of 0.01 ?

(30 pt.)

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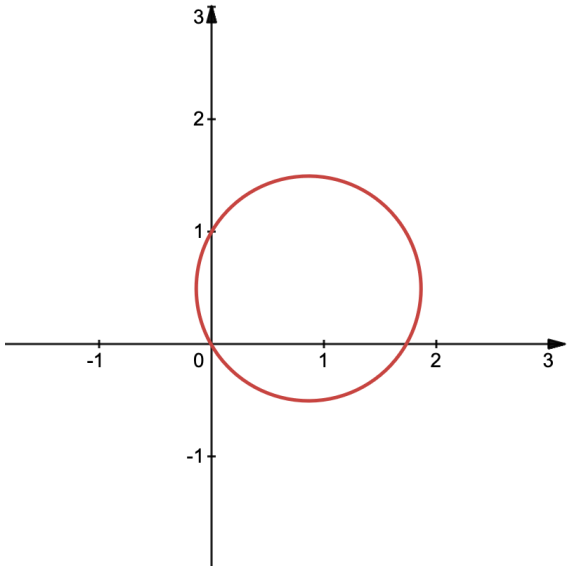
9. Find the area of the segment of the circle with polar equation

$$r(\theta) = \sqrt{3}\cos\theta + \sin\theta; \qquad \theta \in [-\pi/3, 2\pi/3)$$

below the polar axis (in the figure below, the polar axis coincides with the positive X-semiaxis, and the pole with the Cartesian origin).

(20 pt.)



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