Math 125 Final Exam, Spring 2024

Wednesday, May 8th, 2024, 8:00 am–10:00 am

Name: ______________________________

Student ID: ______________________________

Instructions:

- There are 9 problems on this exam.
- Each problem is labeled with its point value.
- There are a total of 100 points available on this exam.
- Please write your name in the designated spot at the bottom of each page.
- You must show all your work and justify your methods to obtain full credit.
- You may use one page of notes: 8.5” $\times$ 11”, hand written on both sides.
- No calculators, or other electronic devices, are permitted. Turn off your cell phone.
- Please simplify answers to a reasonable degree; perform simple algebra and arithmetic. You can leave your answers as sums, differences, products, fractions, powers or roots, e.g. $\frac{12}{13} + 125^2(\sqrt{7})$.
- The last two pages are blank and are intended to be used if you need extra space to solve any of the problems. If you do use these pages to write a solution to one or more of the problems, indicate on those problems that your work appears on the scratch paper pages, and indicate which page.
- All work you submit should represent your own thoughts and ideas. If the graders suspect otherwise, you can expect your instructor to file a report with USC’s Office of Academic Integrity (OAI).

Problem 1. (10 points) Compute the following limits. No credit will be earned if L'Hopital's rule is used.

a.

$$\lim_{x\to\infty} \frac{\sin(x) + 3\cos(x)}{x^2}$$

(Hint: use the squeeze theorem.)

b.

$$\lim_{x\to 0} \left(\frac{1}{x\sqrt{x+1}} - \frac{1}{x} \right)$$

(Hint: combine the function into one fraction and then consider the conjugate.)

 Name:______________________________________

Problem 2. (10 points) Find the values of a and b that make $f(x)$ continuous everywhere. Show all work in finding the values of a and b.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ ax^2 - bx + 3 & 2 \le x < 3 \\ 2x - a - b & x \ge 3. \end{cases}$$

 Name:____

Problem 3. (10 points) Find the derivatives of the functions given below. You do not need to simplify the final form of your derivative.

a.

$$g(x) = \frac{x}{(x^2 + \tan(x))^5}$$

b.

$$f(x) = x^2 \cdot \cos(\ln(x^3 - 1) + 2)$$

.

 Name:____

Problem 4. (10 points) Consider the equation $\cos(xy) = x^2 + y$ and the curve it defines.

a. Find an equation for the tangent line to the curve at the point $(1, 0)$.

b. Use your answer (and the idea of linearization) to find an approximation for a y so that $(1.1, y)$ is on the curve.

 Name:____

Problem 5. (15 points) Consider the function over the interval $(0, 2)$ defined by:

$$f(x) = \frac{1}{x} - \frac{1}{x-2}$$

You may use the following facts:

$$f'(x) = -\frac{1}{x^2} + \frac{1}{(x-2)^2}$$

and

$$f''(x) = \frac{2}{x^3} - \frac{2}{(x-2)^3}.$$

a. What are

$$\lim_{x \to 0^+} f(x) \text{ and } \lim_{x \to 2^-} f(x) \text{ ?}$$

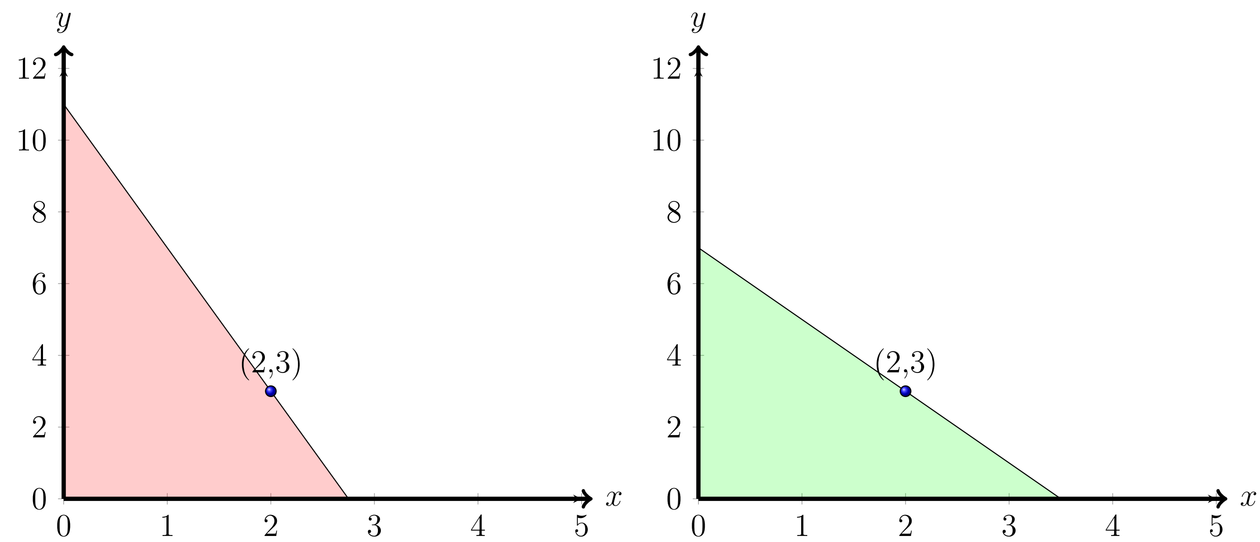
b. Show that f is concave up over the interval $(0, 2)$.

 Name:____

c. Use an appropriate theorem to show that $f'(x) = 1$ has at least one solution over the interval $(0, 2)$.

d. Use part b. to show that $f'(x) = 1$ has at most one solution in the interval $(0, 2)$.

Problem 6. (15 points) A triangle is formed by a line through the point $(2,3)$ and the two positive coordinate axes. Some examples of such triangles are depicted below.



a. Find the point-slope equation for a line through this point $(2,3)$ with slope m.

b. Find the x-intercept of this line in terms of m. (Hint: the x-intercept occurs when $y = 0$.)

c. Find the y-intercept of this line in terms of m.

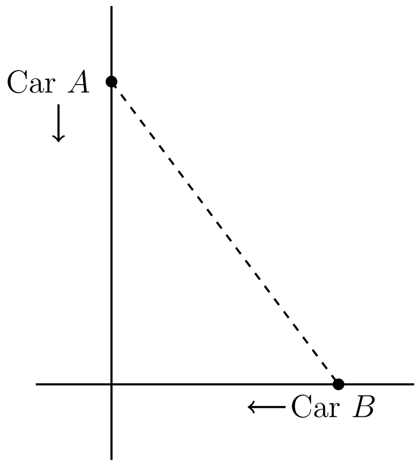
 Name:____

d. Use your answers to the previous two parts to express the area A of this triangle as a function of m. The area of a triangle is $\frac{1}{2} \cdot$ [base] $\cdot$ [height].

e. Find the slope m that minimizes the area of the triangle formed, and justify that your answer indeed produces a minimum area.

 Name:____

Problem 7. (10 points) Two cars are approaching an intersection. One car is approaching from the north and is travelling at 30 mph. The other car is approaching from the east and is travelling at 20 mph. At the moment when the north car is 0.4 miles from the intersection and the east car is 0.3 miles from the intersection, find the rate at which the distance between the two cars is changing.



 Name:____________________________________

Problem 8. (10 points) Evaluate the following integrals.

a.

$$\int \tan(x)\ln(\cos(x))dx$$

(Hint: what is the simplified derivative of $\ln(\cos(x))$?)

b.

$$\int_0^1 \frac{2x+e^{3x}}{3x^2+e^{3x}+3}dx$$

 Name:____

Problem 9. (10 points) Consider the following integral function

$$I(x) = \int_{e}^{e^x} \frac{1}{t^3 + 3} dt$$

a. Find $I'(x)$.

b. Prove that $I(x)$ is an invertible function.

c. Find $(I^{-1})'(0)$.

 Name:____

Scratch Paper Page 13

 Name:____

Scratch Paper Page 14

 Name:____