Math 126: Calculus II — Fall 23

Final Exam — 6 December 2023

First Name: (as in student record)

Last Name: (as in student record)

USC ID: Signature:

- This exam has 9 problems, and will last 120 minutes.
- You may use one page of notes, but no calculator.
- Show all of your work and justify every answer to receive full credit.
- Work quickly, but carefully. Good luck!

Question 1 (20 points). Evaluate the following limits.

(a) $lim_{x\to 0^+}(tan^{-1}(x))^{x^2}$

(b) $lim_{x\to +\infty}(xsin(1/x) - xtan(1/x))$

Question 2 (20 points)**.** Compute the following antiderivatives.

(a) $\int x(x^2+1)ln(x^2+1)dx$

(b) $\displaystyle\int \frac{\sqrt{t^2-9}}{t^6}dt$

Question 3 (15 points)**.** Consider the surface created by revolving the curve $y = e^{-x}$ about the x-axis where $0 \leq x \leq k$.

(a) Write down an integral expressing the area of this surface. **You do not need to evaluate this integral**.

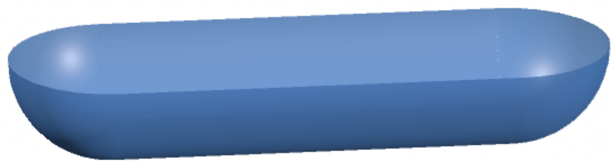
(b) Let $k \to \infty$. Does the improper integral converge? Justify your answer.

Question 4 (20 points).

(a) Let $\mathcal{R}$ be the region bounded by x-axis and the curve $y = e^x + x$, where $0 \leq x \leq 2$. Compute the volume of the solid obtained by rotating $\mathcal{R}$ about the y-axis.

(b) The region $\mathcal{R}$ is bounded by the curves $x = 0$, $y = x^4 + x^2$, and $y = x^2 + 16$. Set up an integral expressing the volume of the solid obtained by rotating $\mathcal{R}$ about $y = -1$. **You do not need to evaluate the integral**.

Question 5 (16 points)**.** A tub filled with water has the shape of a half-cylinder with hemispherical ends, as pictured below. The tub is 5m tall and the base has a length of 20m. Set up an integral expressing the work required to empty the tank by pumping all of the water out of the top. You may assume the density of water is $\rho = 1000\ kg/m^3$ and the acceleration due to gravity is $g = 10\ m/sec^2$. **You do not need to evaluate the integral**.



Question 6 (24 points)**.** Determine whether the following series are conditionally convergent, absolutely convergent or divergent. In each case, be sure to carefully justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \; ln(\tfrac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{2\cdot 5\cdot 8\cdot \ldots \cdot (3n-1)})$

(b) $\displaystyle\sum_{n=4}^{\infty} \frac{(-1)^n \cdot n^{3/2}}{n^3 + n^2 cos(n)}$

$(c)\ \displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{n\sqrt{ln(n^2)}}$

Question 7 (20 points).

(a) Find the interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{(-2x)^n}{n^{3/2} \cdot ln(n+1)}$$

(b) Assume the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ is R, where $a_n \geq 0$. Does the series $\sum_{n=1}^{\infty} a_n \sqrt{n} x^{n-1}$ converge for $0 \leq x < R$? Justify your answer.

Question 8 (10 points).

(a) Find a power series representation of $f(x) = (1 + x)^3 \cdot ln(2 + x)$ and its radius of convergence.

(b) Find $f^{(20)}(-1)$.

Question 9 (20 points). Let $f(x) = x^2 lnx$.

(a) Compute $T_3(x)$ at 1.

(b) What is the maximum possible error in the approximation $f(x) \sim T_3(x)$, when $1/2 \leq x \leq 3/2$.

(Extra page)