MATH 125 FINAL EXAM SPRING 2012

Lytvak at 9AM
Lin at 10AM
Lin at 11AM
Lai at 12PM
Lanski at 1PM

INSTRUCTIONS

Turn off all electronic devices. Calculators, notes, books, or consultation with others is not allowed. Only the proctor may answer questions that arise. **SHOW YOUR WORK** and make your answers to each problem clear. The answers themselves are *not* sufficient for credit. You *must* indicate how you obtained the answer by the procedures you use, or by explanations you make from the course material, and you *must* use *only* the material and methods from this course. When a blank is provided, please put the final answer in the blank. The backs of sheets may be used for scratch paper, but if any part of a solution is on the back of a sheet, then you must indicate that to the grader.

Problem 1 (25 points) _____
Problem 2 (15 points) _____
Problem 3 (30 points) _____
Problem 4 (30 points) _____
Problem 5 (20 points) _____
Problem 6 (25 points) _____
Problem 7 (20 points) _____
Problem 8 (15 points) _____
Problem 9 (20 points) _____
Total _____

1) (25 points) Find the following limits, including $\pm\infty$. Indicate clearly why the limit exists, or why it does not exist.

a) $\lim_{x\to 2} \dfrac{x^2-4}{|x-2|}$

b) $\lim_{x\to\infty} \left(\sqrt{x^4+x^2-5}-x^2\right)$

c) $\lim_{x\to 32} \dfrac{x^{3/5}-8}{x-32}$

d) $\lim_{x\to 0} \dfrac{\sin x}{x+\tan x}$

2) (15 points) Use theorems from the course to argue *carefully* that the equation $x^4 - x - 5 = 0$ has at least two different real solutions (compute some values and apply a theorem to show this) and then argue that it has exactly two real solutions.

3) (30 points) In parts a) – d) find the derivative of the given function. There is a part e).

a) $g(x) = \dfrac{1+\sqrt{x}}{1-\sqrt{x}}$

b) $f(x) = e^{x \cos x}$

c) $y = \int_{x^2}^{x^3} t^2 \sin(t)\, dt$

d) $y = y(x)$ when $ln(y) = y^2 ln(x)$

e) In d), find the equation of the tangent line to the graph of $y(x)$ at the point (1, 1).

4) (30 points) a) Use Calculus to find the linear approximation (i.e. using differentials) of $(999, 900)^{1/6}$. Your answer should be a fraction in lowest terms or a decimal of at least six places.

b) Two ships sail from the same port. The first leaves port at 1PM and travels due east at a speed of 10 miles per hour. The second ship leaves port at 2PM and travels due north at a speed of 15 miles per hour. At what rate is the distance between the ships changing at 3PM? Include the appropriate units.

c) A rectangle is formed inside the region bounded by the y-axis, the line y = 4, and the graph of $y = x^2$ so that one side of the rectangle lies on the y-axis, another side lies on the line y = 4, and one corner is on the graph of $y = x^2$. If $A(x)$ is the area of such a rectangle, then find the minimum and the maximum areas for all such rectangles with $1/2 \le x \le 3/2$.

5) (20 points) Let $g(x) = 1/x - 3/x^3$.

a) Determine all horizontal asymptotes and vertical asymptotes of the graph of $g(x)$.

Horizontal asymptote(s): _____

Vertical asymptote(s): _____

b) On what intervals is $g(x)$ increasing and on what intervals is $g(x)$ decreasing?

Increasing on: _____

Decreasing on: _____

c) For what values of x does $g(x)$ have a local minimum or maximum? Are any absolute? If there are none, write: NONE.

Local min:_____

Local max:_____

Absolute min:_____

Absolute max:_____

d) On what intervals is the graph of $g(x)$ concave up and on what intervals is the graph of $g(x)$ concave down?

Concave up on:_____

Concave down on:_____

e) At what values of x does the graph of $g(x)$ have points of inflection. If there are none, write: NONE.
f) Sketch a rough graph of $g(x)$ using the information above.

6) (25 points) Find the following

a) $\int_0^4 |\sqrt{x} - 1|\, dx$

b) $\int_{-7}^{7} x^{125} e^{x^2}\, dx$

c) $\int \frac{x}{2^{x^2}}\, dx$

d) $\int \frac{x^2}{\sqrt{x+1}}\, dx$

7) (20 points) Write $\lim_{n\to\infty} \frac{\pi}{2n} \sum_{i=1}^{n} \frac{\cos(i\pi/2n)}{1+\sin(i\pi/2n)}$ as a definite integral and then evaluate it.

8) (15 points) a) Carefully state the Mean Value Theorem.
b) If $g(x)$ is continuous and differentiable on the interval (a, b) and if on this interval $g'(x) > 0$ then argue that g must be increasing on (a, b).

9) (20 points) a) Solve for x: $8(2/4^x)^x = 1$
b) Solve for x: $\log_3(2^{\log_5 x}) + \log_3(2^{\log_5(x+1)}) = \log_3 2$
c) For any $a, b > 0$, show that $a^{\ln(b)} = b^{\ln(a)}$
d) For any $a, b > 0$, show that $\log_a(b)/\log_a(b+1)) = \ln(b)/\ln(b+1)$.