MATH 126 **FINAL EXAM** **May 07, 2012**

Last (Family) Name: ____________________ **First Name:**____________________

USC ID: ____________________ **Signature:**____________________

Circle the lecture section you are registered for:

Montgomery at 10   Lin at 10   Lin at 11   Mancera at 11   Lin at 12   Montgomery at 1

INSTRUCTIONS

Show your work to obtain full credit: points may be deducted if you do not justify your answer. You may leave your final answers containing e, π, fractions or $\sqrt{2}$, $\sqrt{3}$, etc., without further simplication. Please indicate clearly whenever you continue your work on the back of the page.

NOT allowed: calculators and other electronic devices, books and lecture notes. Turn off your cell phones.

ALLOWED: one self-prepared handwritten formula sheet, letter size paper, both sides.

Problem	Value	Score
1	20	
2	25	
3	20	
4	15	
5	15	
6	20	
7	20	
8	15	
9	20	
10	25	
Total	200	

1. Find the limits:

(a) $\lim_{x\to 0} \dfrac{\cos(x) - \cos(5x)}{x^2}$

(b) $\lim_{x\to 0^+} (1 + \tan(2x))^{(3/\sin x)}$

(c) $\lim_{n\to\infty} \dfrac{(-3)^n}{n!}$

2. Evaluate the integrals.

(a) $\displaystyle\int_0^1 \ln x \; dx$

(b) $\displaystyle\int \frac{1}{x^4\sqrt{x^2-4}}\,dx$

(c) $\displaystyle\int \frac{x^2+2}{(x^2+x+1)(x-1)}\,dx$

3. Let $\mathcal{R}$ be the region bounded by the curve $y^2 = 4 - x$ and the line $x + y = 2$.

(a) Sketch the region and find its area.

(b) Write an integral for the volume V of the solid obtained by rotating the region $\mathcal{R}$ as in (a) about the line $y = -1$ using the method of cylindrical shells. **Do not evaluate the integral.**

(c) Write an integral for the volume V of the solid obtained by rotating the region $\mathcal{R}$ as in (a) about the y-axis using the slicing method (disks). **Do not evaluate the integral.**

4. Let $f(x) = \ln(\cos x)$, for $0 \le x \le \pi/3$.

(a) Write an integral for the length of the curve $f(x)$ on the given interval. Do not evaluate.

(b) Evaluate your integral in (a).

5. Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{1+x}{xy}$$

for $x > 0$, which satisfies the initial condition $y(1) = -4$.

6. A water tank has the shape of a paraboloid of revolution: its shape is obtained by rotating the parabola $y = x^2/4$, for $0 \leq x \leq 4$, around the y-axis. Assume that the water in the tank is 3 ft deep.

Find the work required to pump water out of the top of the tank, to lower the water level to 2 ft.

(the weight of water is 62.5 lbs/ft^3)

7. In (a), (b), and (c), determine whether each series is conditionally convergent, absolutely convergent, or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n^4-1}}$

(c) $\sum_{n=1}^{\infty} ne^{-n}$

8. Find the sum of the series $\sum_{n=2}^{\infty} \left[\frac{2}{n^2-1} + \left(\frac{2}{3}\right)^n\right]$.

9. Find the radius of convergence and the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+1)^{n+1}.$$

10. In part(a), (b) and (c), you should give the first four non-zero terms in the series explicitly, as well as an expression for the general term.

(a) Find the Maclaurin series for $\dfrac{1}{1+x^2}$. What is the radius of convergence?

(b) Using (a), find the Maclaurin series for $\tan^{-1} x$.

(c) Using (b), find the Maclaurin series for $f(x) = x \tan^{-1}(x^2)$.

(d) Find $f^{(15)}(0)$.

(if you were unable to find the series for part (c), you may do part (d) by assuming that $f(x) = \sum_n b_n x^n$.)

(e) Evaluate $\int_0^1 x \tan^{-1}(x^2)\, dx$ using a series, with an error of at most $\frac{1}{150}$. You may leave your answer as a sum of fractions.