MATH 126 FINAL EXAM SPRING 2011

1) (15 points) Find $\int \sqrt{1-x^2}\, dx$

2) (15 points) Find $\int \dfrac{\arcsin(\sqrt{x})}{\sqrt{x}}\, dx$

3) (20 points) Argue that $\displaystyle\int_{2}^{\infty} \dfrac{x^2-4x-1}{(x^2-1)(x^2+1)}\, dx$ diverges, or find its value.

4) (15 points) Argue that $\displaystyle\int_{0}^{\pi} \dfrac{e^{-x^2}}{x}\, dx$ diverges, or find its value.

5) (20 points) This problem has three parts.
 A) Let R be the bounded region of the plane enclosed by the x-axis and graph of $y = 3x - x^2 - 2$.
For each part below find an integral that gives the required answer: DO NOT EVALUATE THE INTEGRALS.

        i) The volume of the solid obtained by rotating R about the x-axis.

        ii) The volume of the solid obtained by rotating R about the line $x = -3$.

B) A plate in the shape of a symmetric trapezoid three meters wide at the bottom, five meters wide at the top and two meters high (a rectangle with congruent triangles on each side:

is submerged vertically with its top at the surface of a liquid of density ρ. Find an integral that gives the hydrostatic force on one side of the plate: DO NOT EVALUATE THE INTEGRAL. Use g for the acceleration due to gravity.

6) (20 points) In each case below, the n-th term of a sequence is given. Give reasons why the sequence diverges, or why it converges and then find its limit.

       i) $a_n = \left(\dfrac{4n-3}{n} - \dfrac{3n}{n+1} \right)^{n^2+n}$

       ii) $b_n = (3 + \sin(n))^{1/n}$

       iii) $c_n = (-1)^n(1 - (1/n))$

7) (20 points) In each case give reasons why the series converges or why it diverges. Each series begins at $n = 1$.

       i) $\displaystyle\sum \left(\dfrac{3}{5}\right)^{n+1} \left(\dfrac{7}{5}\right)^{n}$
   ii) $\displaystyle\sum \dfrac{\sin(1/n)}{\tan(1/n)}$
   iii) $\displaystyle\sum \dfrac{e^{1/n}-1}{n}$ (compare to $1/n^2$)

8) (20 points) Find the radius of convergence and the interval of convergence of

$$\sum_{1}^{\infty} \frac{(-1)^n (x+1)^n}{n5^n}.$$ Give reasons for your answers.

9) (15 points) Find $T_3 (x)$, the third Taylor polynomial of $g(x) = x^{4/3}$ about 8.
   Use $T_3(x)$ to approximate $7^{4/3}$ as a sum of fractions.
   Using the Taylor remainder R_3, what substitution in it gives the best estimate for the error in the approximation above.

10) (15 points) Find the Taylor series about 0 (the Maclaurin series) for $g(x) = xe^{-x^2}$, then find $g^{[17]}(0)$ and $g^{[20]}(0)$.

11) (25 points)
   i) (10 points) What integral gives the length of the graph of $r = 1/\theta$ from $\theta = \pi$ to $\theta = 2\pi$.
   ii) (15 points) Integrate the function that appears in this definite integral, OR (not both)

   (10 points) Find $\int \frac{x^5}{\sqrt{1+x^2}} dx$