

Last Name _____ First Name _____

USC ID: _____ Signature _____

Circle the lecture section you are registered for:

Kamienny at 11am

Yin at 9am

Kamienny at 12pm

Yin at 10am

Instructions

Read the problems carefully and answer the questions asked. Write neatly and indicate clearly your answer to each problem. The backs of the sheets may be used for scratch paper or to continue your work on a problem, but if you do continue your answer, please give directions to the grader.

You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Calculators, notes, books, or collaboration with others are it not allowed. If you have any questions about any of the problems ask the proctor, but no one else!

Problem	Points	Score
1	24	
2	28	
3	40	
4	24	
5	16	
6	18	
7	16	
8	18	
9	16	
Total	200	

1. Find the following limits. Do not use L'Hôpital's rule.

a) $\lim_{x \rightarrow 1^+} \frac{x^2 - 9}{x^2 + 2x - 3}$

b) $\lim_{t \rightarrow 0} \frac{t^3}{\tan^3 2t}$

c) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + ax} - \sqrt{x^2 + bx})$, where a and b are two constants.

2. Differentiate the following functions.

a) $y = \int_{\cos x}^2 \sqrt{1+t^4} dt.$

b) $y = (\cos x)^x.$

c) $h(t) = \frac{3t - 2}{\sqrt{2t + 1}}$

d) Find $\frac{dy}{dx}$ if $x^2 \cos y + \sin 2y = xy$.

3. Consider the function $y = \frac{x-1}{x^2}$. Note that $dy/dx = 2/(x^3) - 1/(x^2)$, and $d^2y/dx^2 = 2/(x^3) - 6/(x^4)$.

a) Find the domain.

b) List x -intercepts and y -intercepts, if any.

c) Is this function symmetric (even or odd)? Why?

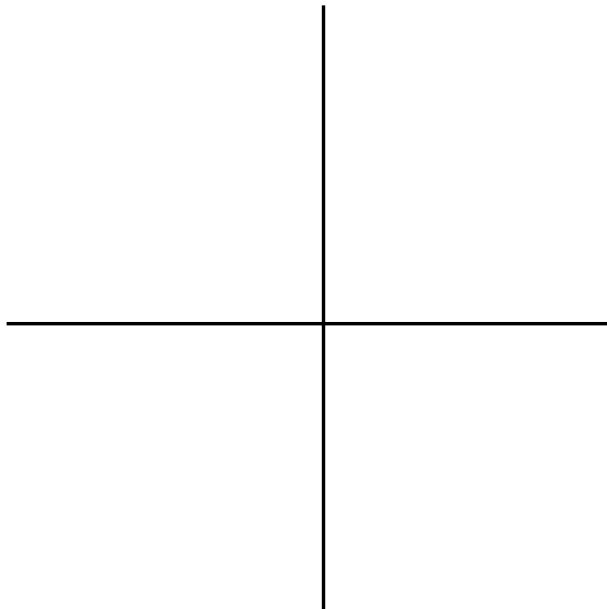
d) Find all asymptotes, if any.

e) Find the intervals of increase or decrease.

f) Find all local maximum and minimum values.

g) State the concavity of the function. Find points of inflection.

h) Sketch the curve.



4. Find the volume of the largest circular cone that can be inscribed in a sphere of radius r .

[*hint*: Set the base radius of the cone to be x , and set the distance from the center of the sphere to the base of the cone to be y . Then the height of the cone is $y + r$. The volume of the cone is $V = \frac{1}{3}\pi x^2(y + r)$].

5. Find the area bound by the graph of $y = x^3 - x$ and the x -axis.

6. A 500 gm. sample of a radioactive substance decays so that after 2 years only 350 gms. remain.

a) Find a formula for the amount of the substance present at time t .

b) What is the half-life of the substance?

c) How much of the substance is left after five years.

- Find the linear approximation of the function $y = (1 + x)^{1/2}$ at $a = 0$, and use it to approximate $(.98)^{1/2}$.

8. Compute the integrals

a) $\int \sin x / (1 + \cos x) dx$

b) $\int x^3 \cdot \sqrt{x^2 + 1} dx$

c) $\int_1^4 (x^2 - 3\sqrt{x})/x dx$

9. Write the limit $\lim_{n \rightarrow \infty} (2/n) \cdot \sum_{i=1}^n (1 + i/n)^{3/2}$ as an integral, and compute the integral.