MATHEMATICS 126 **FINAL EXAM** **May 10, 2010**

Last Name (Print)_____ First Name (Print)_____

USC ID Number _____Signature _____

Circle your Professor's Name

Daoust-Ritz (MWF 1PM) **He** (MWF 11AM) **Mancera** (MWF 10AM)

Montgomery (MWF 10AM) **Sakai** (MWF 12 NOON)

Directions: **You must show all your work and justify your methods to obtain full credit**.

You can leave expressions in terms of radicals, powers, exponentials and logarithms such as $\ln 5$, e^5, $\sqrt{3}$. Do not use scratch paper. Use the back side of the previous page if additional room is needed. Write your answers in the appropriate places.

No calculators are allowed (or needed). However, you can use a two-sided formula sheet. All cell phones and pagers must be in your backpack, put in the "silent" mode.

Remember USC considers cheating to be a very serious issue.

Do not write on this page below this line.

1. (20)	
2. (30)	
3. (20)	
4. (20)	
5. (15)	
6. (20)	
7. (40)	
8. (20)	
9. (15)	
Total (200)	

1. (20 points) Find the following limits if they exist

a) (10 points) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} =$

b) (10 points) $\lim_{x\to 0} (\sin(x))^x$

$\lim_{x\to 0} (\sin(x))^x =$

2. (30 points) Compute the following integrals

a) (10 points) $\int_1^2 (\ln(x))^2 dx$

$\int_1^2 (\ln(x))^2 dx =$

b) (10 points) $\int \tan^3(x)\sec^4(x)dx$

Problem 2 continued.

$$\int \tan^3(x)\sec^4(x)\,dx =$$

c) (10 points) $\displaystyle\int \frac{2x^2+x+2}{x(x^2+1)}\,dx$

$$\int \frac{2x^2+x+2}{x(x^2+1)}\,dx =$$

3. (20 points) Let $\mathfrak{R}$ be the region of the plane bounded by the line $y = x - 1$ and the curve $x = 1 + y^2$.

a) (10 points) Sketch the region and find its area.

Area of $\mathfrak{R}$ =

For b) and c) consider the solid of revolution S obtained by rotating $\mathfrak{R}$ about the y-axis.

b) (5 points) Write an explicit integral for the volume V of S using the method of slicing (disks). DO NOT EVALUATE THE INTEGRAL.

c) (5 points) Write an explicit integral for the volume V of S using the method of cylindrical shells. DO NOT EVALUATE THE INTEGRAL.

4. (20 points) For each of the following integrals, determine whether they converge or diverge.

(Remember: **You must show all your work and justify your answers to get full credit**)

a) (10 points) $\int_1^5 \frac{y}{y-1}\,dy$

Circle your answer:
Convergent Divergent

b) (10 points) $\int_1^\infty \frac{\sin^2(x)}{x^3+1}\,dx$

Circle your answer:
Convergent Divergent

5. (15 points) A tank has a (truncated) conical shape obtained by rotating $y = 5(x-3), 0 \le y \le 5$ about the y-axis, where x and y are measured in meters. Water is filled in the tank to a height of $y = 4m$. Set up, but DO NOT EVALUATE, the integral that expresses the work needed to pump all the water to the top of the tank ($y = 5m$). Use $\rho\left[kg/m^3\right]$ for the water density and $g\left[m/s^2\right]$ for the acceleration of gravity.

Work =

6. (20 points) Determine whether each of the following sequences is convergent or divergent. Find the limit, if the sequence is convergent.

(Remember: **You must show all your work and justify your answers to get full credit**)

a) (10 points) $a_n = \dfrac{(3\cos(n))^n}{5^n + 2^n}$

Circle your answer:	
Convergent	Divergent

b) (10 points) $a_n = (-1)^{n+1} \dfrac{4n^2 + 1}{5n - n^2}$

Circle your answer:	
Convergent	Divergent

7. (40 points) Establish convergence or divergence.

(Remember: **You must show all your work and justify your answers to get full credit**)

a) (10 points) $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

Circle your answer:	
Convergent	Divergent

b) (10 points) $\sum_{n=1}^{\infty} \frac{1}{e^n - n^e}$

Circle your answer:	
Convergent	Divergent

Problem 7 continued.

c) (10 points) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n+3}}$

d) (10 points) Find the radius of convergence R, and the interval of convergence for the series: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3^n x^n}{n}$

R =	Interval of convergence =

8. (20 points) Let $f(x) = x^2 e^{-x^2}$.

a) (10 points) Find the Maclaurin series of $f(x)$ and its radius of convergence.

$f(x) = x^2 e^{-x^2} =$ R=

b) (5 points) Find $f^{(20)}(0)$.

$f^{(20)}(0) =$

c) (5 points) Evaluate $\int_0^1 f(x)dx$ correct to within 0.01. (you can leave your answer as a sum).

$\int_0^1 f(x)dx =$

9. (15 points) Consider the circles $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$.

a) (7 points) Find their corresponding Cartesian equations and sketch the circles.

b) (8 points) Find the area that lies inside both of the circles.

Area =