MATH 126 **FINAL EXAM** **May 11, 2009**

Last Name: _____ **First Name:** _____

USC ID: _____ **Signature:** _____

CIRCLE YOUR LECTURE SECTION (Professor and time):

Lanski at 10 Wang at 10 Mikulevicius at 11
Sakai at 12 Bene at 12 Montgomery at 1

INSTRUCTIONS

Calculators, notes, books, or consulting others are **not** allowed. If you have any questions, ask the proctor, but no one else! Indicate your answers clearly. The backs of the sheets may be used for scratch paper or to continue your work. If you wish to have such work counted, please **give directions** to the grader.

SHOW YOUR WORK to obtain full credit: points may be deducted otherwise.

Problem	Value	Score
1	20	
2ab	20	
2c	10	
3	15	
4	15	
5	15	
6	30	
7	15	
8ab	10	
8cd	10	
9	20	
10	20	
Total	200	

1. (20 pts) Find the following limits:

   (a) $\lim_{x\to 1} \dfrac{\sin^2(\pi x)}{(x-1)^2}$

   (b) $\lim_{x\to\infty} \left(\dfrac{x+3}{x}\right)^{x+1}$

   (c) $\lim_{n\to\infty} \dfrac{1}{1+\frac{3}{4}+\frac{9}{16}+\cdots+(\frac{3}{4})^n}$

2. (30 pts) Evaluate the following integrals:

(a) $\displaystyle\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$

(b) $\displaystyle\int e^{2x}\sin(e^x)\,dx$

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(c) $\displaystyle\int_0^\infty \frac{dx}{x^2+3x+2}$

3. (15 pts) Consider the region $\mathcal{R}$ in the plane bounded by $y = x - x^2$ and $y = x - 1$. Let $\mathcal{S}$ be the solid obtained by rotating $\mathcal{R}$ around the line $x = 2$. Find an integral which expresses the volume of the solid $\mathcal{S}$. DO NOT EVALUATE your integral.

4. (15 pts) A tank has the shape obtained by rotating about the y-axis the region bounded by $y = x^3$, $x = 0$, $y = 1$, and $y = 8$, where x and y are measured in meters. Assume that the tank is filled with liquid. Find an integral which expresses the work done in pumping all of the liquid out of the top of the tank.

   Include a sketch of the tank. DO NOT EVALUATE your integral.

   (use ρ [kg / m^3] to denote the density of the liquid and g [m/s^2] to denote the acceleration of gravity).

5. (15 pts) A plate in the shape of an equilateral triangle with side length 2 feet is suspended vertically in a liquid. The side closest to the surface is parallel to the surface and 2 feet below it. Find an integral which expresses the force of the hydrostatic pressure of the liquid on the side of the plate. DO NOT EVALUATE your integral.

   (use δ [lb / ft^3 to denote the weight density of the liquid).

6. (30 points) Determine if the following series converge or diverge. State clearly any test(s) you use.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{n}$.

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}\ e^{\sqrt{n}}}$

(c) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{n}{3n+2}\right)^n$

7. (15 points) Find the interval and radius of convergence of $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n(-9)^n}$

8. (20 points) You may assume that $\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$, for $|x| < 1$.

(a) Find the Maclaurin series for $f(x) = \dfrac{\ln(1+x^4)}{x^2}$.

(b) Write out the three first terms of your series in (a), and find $T_7(x)$, the seventh degree Taylor polynomial for the function.

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(c) What is $f^{(18)}(0)$?

(d) Evaluate $\int_0^{0.5} f(x)dx$ as a power series.

9. (20 pts) (a) Approximate $f(x) = x^{1/3}$ by its Taylor polynomial of degree 3 at $a = 1$.

(b) Use Taylor's formula to estimate the accuracy of this approximation at $x = 0.5$ (it is not necessary to simplify your answer).

10. (20 points) Sketch the curve $r = \cos^2(\theta)$ and then find the area enclosed by the curve.