MATH 126 **FINAL EXAM** **May 12, 2008**

Last Name: _____ **First Name:** _____

USC ID: _____ **Signature:** _____

CIRCLE YOUR LECTURE SECTION (Professor and time):

Montgomery at 10 Raskind at 10 Adeboye at 11
Lototsky at 12 Emerson at 12 Kamienny at 1

INSTRUCTIONS

Read the problems carefully and answer the questions asked. Write neatly and indicate clearly your answer to each problem. The backs of the sheets may be used for scratch paper or to continue your work on a problem, but if you do continue your answer, please give directions to the grader.

You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Calculators, notes, books, or collaboration with others are *not* allowed. If you have any questions about any of the problems ask the proctor, but no one else!

Problem	Value	Score
1	20	
2	15	
3	20	
4	20	
5	20	
6	20	
7	20	
8	10	
9	10	
10	25	
11	20	
Total	200	

1. (20 pts) Evaluate the following integrals:

(a) $\displaystyle\int \frac{x\,dx}{\cos^2 x}$

(b) $\displaystyle\int \frac{dx}{x^4 - 16}$

2. (15 points) Does the integral $\int_0^1 \frac{\cos(2x)-\cos(3x)}{x^2}\,dx$ converge or diverge?

3. (20 points) Find the following limits:

   (a) (10 points) $\lim_{x\to\infty} e^{-x}\sin x$

   (b) (10 points) $\lim_{x\to\infty}(3+x)^{1/x}$

4. (20 points) The base of a solid S is a disk of radius one. Parallel cross sections perpendicular to the base are equilateral triangles.

   SET UP AN INTEGRAL FOR THE VOLUME OF THE SOLID, BUT DO NOT EVALUATE IT.

5. (20 points) A water trough has a rectangular top and semicircular ends (that is, the shape of half of a right circular cylinder, cut in half lengthwise). The ends each have diameter 4 meters, and the sides of the trough are each 10 meters long. Assuming that the tank is full of water, how much work is done in pumping the water out over the top of the tank so that the water left is 1 meter deep? Please use that the density of water is 1000kg per cubic meter and call the acceleration due to gravity g.

   SET UP AN INTEGRAL FOR THE WORK DONE, BUT DO NOT EVALUATE IT.

6. (20 points) Find the solution $y(x)$ of the differential equation:

$$\frac{dy}{dx} = 5xe^{-x^2}(y-1)$$

that satisfies $y(1) = 2$.

7. (20 points) Determine if the following series converge or diverge. Be sure to clearly state any test(s) you use. If a series converges, does it converge absolutely?

(a) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$.

8. (10 points) Compute the value of $\sum_{n=1}^{\infty} 2^{-n-1}(\sqrt{3})^{-n}$.

9. (10 points) Determine the interval of convergence of $\sum_{n=2}^{\infty} \frac{(-2)^n x^n}{n}$

10. (25 points) Throughout this problem, let $g(x) = x^4 e^{-x^2}$.

(a) Find the Taylor series about $a = 0$ for $g(x)$

(b) What is $g^{(84)}(0)$?

(c) What is the radius of convergence of the series in part (a)?

(d) Use part (a) to approximate $\int_0^1 g(x)\,dx$ to within 10^{-3}

11. (20 points) Sketch the curve and find the area enclosed by $r = 2 + \cos 2\theta$