MATH 125 FINAL EXAMINATION
May 7th 2007

Last Name: _____ **First Name:** _____

Student ID Number: _____ **Signature:** _____

Please circle the class in which you are registered:

Haskell (9am) Malikov (10am) Haskell (11am) Proskurowski (12pm)

Directions: Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. You may not use a calculator or notes of any kind. The exam is worth a total of 400 points.

Problem	Points	Score
1	40	
2	30	
3	40	
4	40	
5	50	
6	50	
7	40	
8	50	
9	30	
10	30	
Total	400	

Problem 1. Compute the following limits if they exist. They may be finite or infinite. You may **not** use L'Hôpital's rule, if you know what that is. You must justify your answers.

a) $\lim_{x \to -4} \dfrac{x^2 + x - 12}{x^2 + 5x + 4}$

b) $\lim_{x \to 2} \dfrac{x^2 - 4}{\sin(x - 2)}$

c) $\lim_{x \to 0^-} e^{1/x} \cos x$

d) $\lim_{n \to \infty} \dfrac{5n(n+1)(2n+1)}{6n^3}$

Problem 2. Consider the function

$$f(x) = \begin{cases} 2x & x \leq 1 \\ x^2 + 3 & x > 1 \end{cases}$$

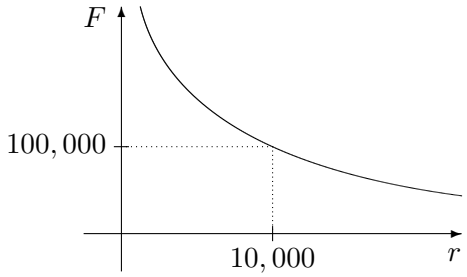
a) Is f continuous at $x = 1$? Justify your answer.

b) Find $\lim_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$.

c) Find $\lim_{x \to 1^-} \dfrac{f(x) - f(1)}{x - 1}$.

d) Is f differentiable at $x = 1$? Justify your answer.

Problem 3. Let F denote the gravitational force (measured in Newtons) exerted by the earth on a rocket and let r denote the distance of the rocket from the center of the earth (measured in kilometers). Notice that F is a function of r as shown in the sketch below.



Suppose when $r = 10,000$ that $F = 100,000$ and $dF/dr = -10$.

a) Estimate the value of F when $r = 11,000$. Is your estimate an underestimate or an overestimate? Justify your answer.

b) Suppose the rocket is moving away at a speed of 0.2 km/sec, i.e.

$$\frac{dr}{dt} = 0.2.$$

How fast is the gravitational force changing when the rocket is 10,000 km away from the center of the earth?

Problem 4. Consider the curve given by the equation

$$x \ln y + y = 1 - \ln x \,.$$

Notice that the point $(1, 1)$ lies on this curve.

a) Find the equation of the tangent line to the curve at the point $(1, 1)$.

b) Estimate the y-coordinate of that point on the curve that is close to $(1, 1)$ and whose x-coordinate is equal to 1.2.

Problem 5. Consider the function

$$f(x) = \frac{1}{x} - \frac{1}{x+1}\,.$$

Fill in all of the information below. Don't leave any lines blank; write N/A if the function doesn't have the property indicated. Then, sketch the graph of the function. All of the information should be clearly visible on your graph.

a) Domain of f: _____

b) Location of any x-intercepts: _____

c) Location of any vertical asymptotes: _____

d) Location of any horizontal asymptotes: _____

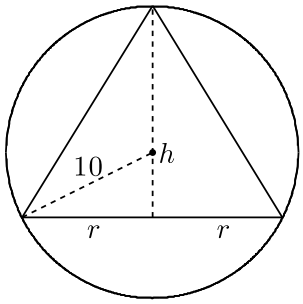
e) The x-coordinates of any local minima: _____

f) The x-coordinates of any local maxima: _____

g) All intervals where the function is concave up: _____

h) The x-coordinates of any points of inflection: _____

Problem 6. A cone of height h cm and radius r cm is inscribed in a sphere of radius 10 cm so that its vertex and the boundary of its base belong to the sphere. The picture below shows a cross-section through the center of the sphere. Find h so that the volume of the cone is maximal.



Please note, the volume of a cone of radius r and height h is $(1/3)\pi r^2 h$.

Problem 7. Consider the equation

$$x^5 + 2x^3 + 4x - 10 = 0\,.$$

a) Use the Intermediate Value Theorem to show that the equation has at least one solution.

b) Use the Mean Value Theorem or Rolle's theorem to show that the equation has at most one solution.

Problem 8. Evaluate the following integrals:

a) $\displaystyle\int \frac{dx}{\sqrt{2-3x}}$

b) $\displaystyle\int_1^4 \frac{x^2-x+1}{\sqrt{x}}\,dx$

c) $\displaystyle\int_0^1 \sin \pi x\,dx$

d) $\displaystyle\int_0^1 \frac{e^x}{5e^x-3}\,dx$

e) $\displaystyle\int \frac{\ln x}{x}\,dx$

Problem 9. Find the derivative of each function. Simplify your answers.

a) $f(x) = \ln(xe^x)$

b) $f(x) = \ln \sqrt{\dfrac{x+1}{x-1}}$

c) $f(t) = \sin(\ln t)$

Problem 10. A sample of radon-222 decayed to 58% of its original amount in 3 days.

a) What is the half-life of radon-222?

b) How long does it take a sample of radon-222 to decay to 10% of its original amount?