MATH 126 **FINAL EXAM** **May 7, 2007**

Last Name: _____ **First Name:** _____

Student ID Number: _____ **Signature:** _____

Circle your instructor's name:

Bene (10 am) Bene (11 am) Honda Lanski Mancera Mikulevicius

INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of 200 points.

Problem	Value	Score
1	10	
2	40	
3	15	
4	20	
5	15	
6	10	
7	10	
8	20	
9	30	
10	10	
11	20	
Total	200	

1. (10 points) Find the limit if it exists:

$$\lim_{x \to 0} \frac{e^x - 1 - x}{\cos(2x) - \cos(3x)}$$

2. (40 points) Evaluate the following integrals.

   (a) $\int x \tan^2(x)\, dx$

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(b) $\displaystyle\int \frac{2}{x^4 + x^2}\, dx$

(c) $\displaystyle\int \frac{x^3}{\sqrt{1 - x^2}}\, dx$

3. (15 points) Evaluate the integral $\int_0^1 \ln x \; dx$.

4. (20 points) Consider the region $\mathcal{R}$ bounded by $y = x^2 - 6$ and $y = x$. Set up, *but do not evaluate*, an integral for the volume of the solid obtained by rotating the region $\mathcal{R}$

   (a) about the line $y = 6$.

   (b) about the line $x = -6$.

5. (15 points) A tank has a shape of a hemisphere of radius 6 m, its flat face on the bottom. It is filled to half its height with water. Set up, *but do not evaluate*, an integral for the work required to empty the tank by pumping water to the top of the tank. Use the fact that the acceleration due to gravity is $g = 9.8$ m/s^2 and the density of water is ρ=1000 kg/m^3.

6. (10 points) Find the length of the curve

$$y = \frac{2}{3}(x-1)^{3/2}, \quad 5 \leq x \leq 8.$$

7. (10 points) Does the sequence converge? If so, find its limit:

$$a_n = \left(\frac{n+1}{n-1}\right)^n$$

8. (20 points) Find the radius of convergence and the interval of convergence of

$$\sum_{n=2}^{\infty} \frac{(x+2)^n}{\ln n}$$

9. (30 points) Consider the function $g(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$

(a) Write the Maclaurin series for $g(x)$.

(b) Find $g^{(10)}(0)$.

(c) Use the Maclaurin series to approximate

$$\int_0^{1/2} g(x)\ dx$$

to within 0.001. Leave your answer as a fraction.

10. (10 points) Does the series converge or diverge? Find the sum of the series if it converges:

$$\sum_{n=1}^{\infty} \left(\frac{11}{4}\right)^n \left(\frac{3}{5}\right)^{2n}$$

11. (20 points) Find the polar equations of the circles of radius 1 with centers at $(0, 1)$ and at $(1, 0)$. Using these equations, find the area of the region that lies inside both circles.