MATH 125 FINAL EXAM 8 May 2006

Last Name: ______________________ **First Name:** ______________________

Student ID Number: ______________________ **Signature:** ______________________

Circle your professor's name:

Haydn Mancera (9am) Mancera (10am) Montgomery

INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Marks may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of 200 marks.

Problem	Value	Score
1	20	
2	18	
3	20	
4	20	
5	24	
6	18	
7	20	
8	20	
9	20	
10	20	
Total	200	

1. (20 marks) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$. Draw the ellipse and locate the solution points. You must verify that your solution points correspond to a maximum of an appropriate function.

2. (6 marks each) Find the limits, if they exist (use any method, except l'Hôpital's rule):

(i) $\lim_{x \to -2} \frac{x+2}{x^2 - 8x - 20}$

(ii) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

(iii) $\lim_{x \to \infty} \left(\ln(1+2x) - \ln(1+x) \right)$

3. (10 marks each) (i) Evaluate the limit $\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$

(Hint: Express the limit as an integral over $[0, 1]$).

(ii) Differentiate $g(x) = \int_1^{\cos x} \ln t \, dt, \quad 0 < x < \frac{\pi}{2}$

4. (10 marks each) Consider the function $f(x) = \ln x - (x-4)^2$.

(i) Using the Intermediate Value theorem, show that the function has at least one root in the interval $[1, 3]$.

(ii) Show that $f(x)$ has exactly one root in $[1, 3]$.

5. (4 marks each) Let $f(x) = \dfrac{x^2 - 3}{x^3}$ (and note that its first derivative is $f'(x) = \frac{9-x^2}{x^4}$).

(i) horizontal or vertical asymptotes: ________________________________

(ii) intervals where increasing________________ or decreasing________________

(iii) local maxima or minima____________________________________

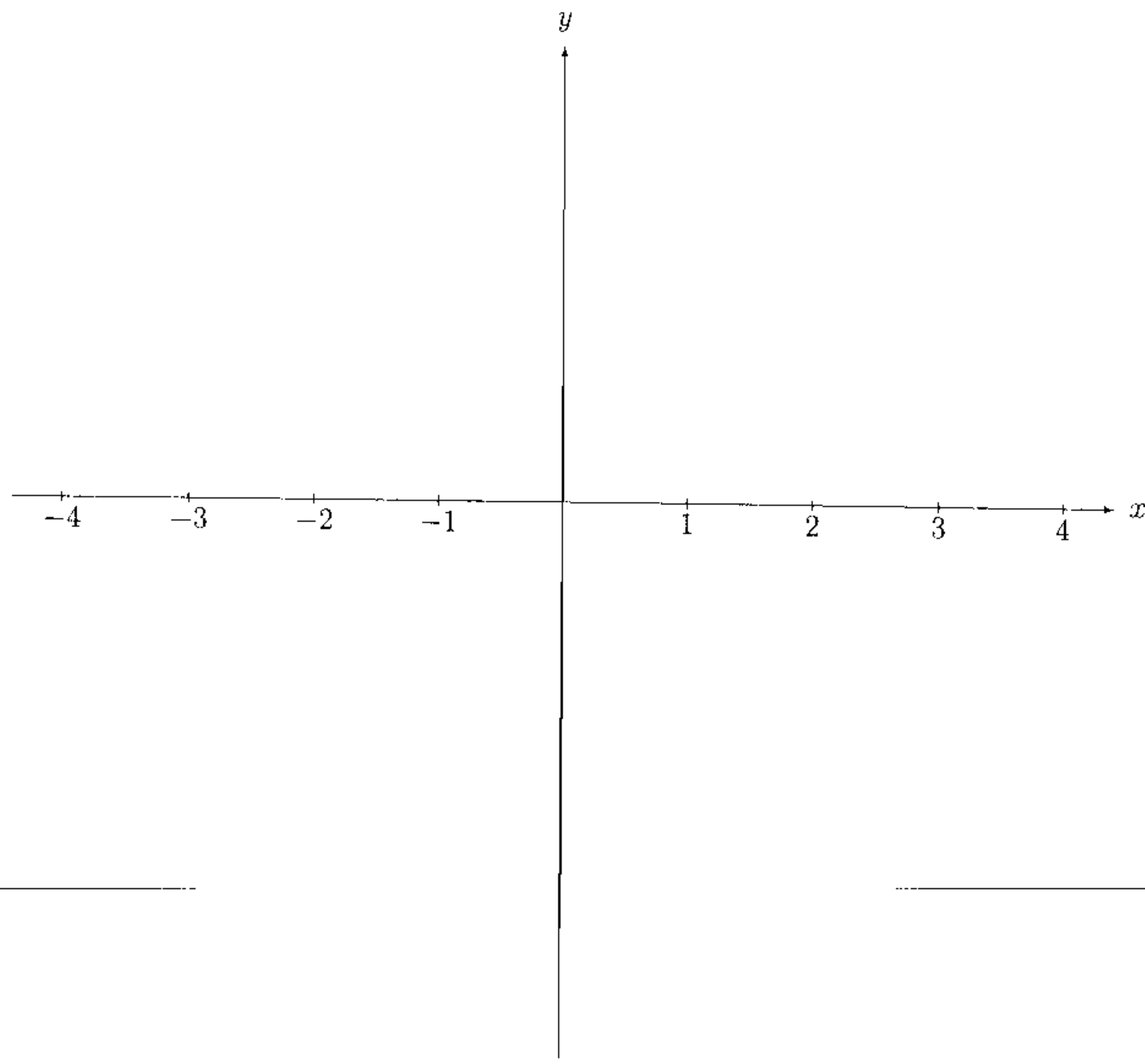
(iv) intervals where concave up________________ or down________________

(continued on the next page)

(continuation of problem 5)

(v) points of inflection____________________

(vi) sketch the graph



6. (6 marks each) Differentiate

(i) $f(x) = x^{\sqrt{x}}$.

(ii) $y = t^2 e^{-t}$

(iii) $w = \dfrac{1}{\sqrt[3]{1 + \cos 2\vartheta}}$

7. (10 marks each) Let C be the curve $xy + y^2 = 1$.

(i) Find the equation of the tangent at $(0, -1)$.

(ii) Find $\dfrac{d^2y}{dx^2}$ at the point $(0, -1)$.

8. (20 marks) Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's radius increases at a constant rate. (A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.)

9. (10 marks each) Evaluate the following integrals:

(i) $\int_0^1 t\,10^{-t^2}\,dt$

(ii) $\int_e^{e^4} \frac{dt}{t\sqrt{\ln t}}$

10. (10 marks each) Integrate:

(i) $\int \frac{x}{\sqrt{1+2x}}\,dx$

(iv) $\int 3\cos\frac{\vartheta}{5}\,d\vartheta$