MATH 126 **FINAL EXAM** **May 8, 2006**

Last Name: ________________ **First Name:**________________

Student ID Number: ________________ **Signature:**________________

Circle your instructor's name:

Bene Honda Kamienny Mikulevicius Sadhal Ziane

INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of 200 points.

Problem	Value	Score
1	30	
2	20	
3	20	
4	20	
5	20	
6	20	
7	10	
8	20	
9	20	
10	20	
Total	200	

1. (10 points each) Evaluate the following integrals.

   (a) $\int x^2 e^{3x}\, dx$

   (b) $\int \frac{8x - 11}{x^2 - 3x + 2}\, dx$

(Continued on the next page.)

(Continued from the previous page.)

(c) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

2. (10 points each) Is the integral convergent or divergent? Carefully justify your answer.

(a) $\displaystyle\int_1^\infty \frac{\sin^2 x}{1+x^3}\, dx$

(b) $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\, dx$

3. (20 points) Let $\mathcal{R}$ be the region bounded by $y = x^2$, $x = 1$, and $y = 0$, and let S be the solid of revolution obtained by revolving $\mathcal{R}$ about the line $x = 2$. Write, but *do not evaluate*, an integral that represents the volume of S. *You must use the method of cylindrical shells.*

4. (20 points) Consider a bowl obtained by rotating the curve $y = x^2$, $0 \leq x \leq 16$, about the y-axis (units are in meters). The bowl is filled with water to a height of $y = 9$ m. Set up, but *do not evaluate*, an integral which gives the work required to pump all of the water to the top of the bowl ($y = 16$ m). Be sure to define all variables. Use ρ to denote the density of water, and g to denote the acceleration due to gravity.

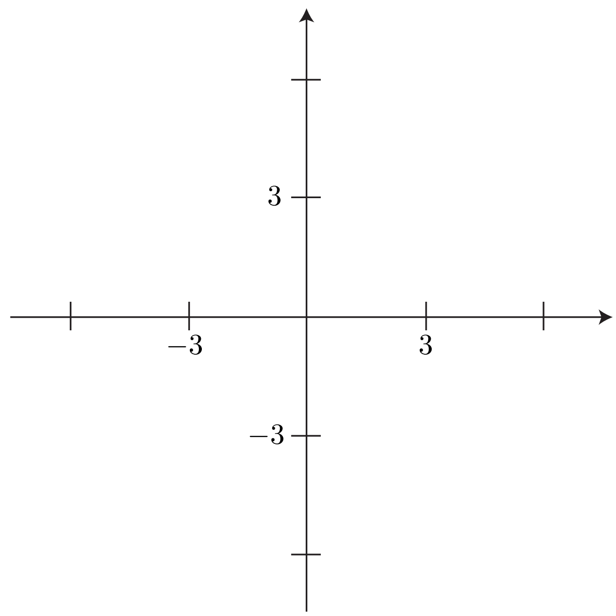
5. (10 points each) Is the series convergent or divergent? Carefully justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n} + 3\sqrt{n^3}}{(2n)^2}$

(b) $\displaystyle\sum_{n=5}^{\infty} \frac{1}{n \ln n}$

6. (20 points) Consider the curve with polar equation $r = \theta$, $0 \leq \theta \leq 2\pi$.

(a) Sketch the curve.



(b) Write, but *do not evaluate*, an integral that represents the length of the curve.

7. (10 points) Find $\lim_{n\to\infty} (1/n)^{1/n}$.

8. (20 points) Find the radius of convergence *and* the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2^n(n+1)}$.

9. (20 points) Consider the function $f(x) = x^4 \cos(x^5)$.

(a) Find the Taylor series for $f(x)$ centered at $a = 0$. Express your answer in *summation notation.* (Hint: Use the Taylor series of $\cos x$ centered at $a = 0$.)

(b) What is $f^{(44)}(0)$?

10. (20 points) Let $f(x) = \int_0^x \sqrt{1+t^4}\, dt$.

(a) Find the first 3 non-zero terms in the Taylor series for $f(x)$ centered at $a = 0$. (Hint: Use the Binomial Series.)

(b) Find an upper bound for the error if we approximate $f(1)$ using the degree 8 Taylor polynomial centered at $a = 0$.