1. (6 points each) Calculate the following limits.
   a) \( \lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} \).
   b) \( \lim_{x \to 0} \frac{1 - \sqrt{1 - x^2}}{x^2} \).
   c) \( \lim_{x \to 8} \frac{|x - 8|}{x - 8} \).
   d) \( \lim_{x \to 2^+} \frac{e^{1/2}}{e^{x/2}} \).

2. (6 points each) Find \( \frac{dy}{dx} \).
   a) \( y = x^2 \sin 2x \).
   b) \( y = \frac{(x - 1)(x - 4)}{(x - 2)(x - 3)} \) (You may use logarithmic differentiation).
   c) \( xe^{-y} + ye^{-x} = 3 \).
   d) \( y = \int_{\frac{1}{3}}^{\sqrt{x}} \cos \frac{t}{t} dt \).

3. (7 points each) Evaluate the following integrals:
   (a) \( \int \left( \sqrt{x} - x^3 + \frac{1}{\sqrt{x}} \right) dx \).
   (b) \( \int_0^1 \frac{5x^2}{x^3 + 2} dx \).
   (c) \( \int_0^\pi x \cos(\pi + x^2) dx \).
   (d) \( \int_0^e 2x \sqrt{1 + e^x} dx \).

4. Consider the following function and its first and second derivative:
   \( f(x) = \frac{x^2 + 3x + 1}{x^2 + 1} \quad f'(x) = \frac{3(1 - x^2)}{(x^2 + 1)^2} \quad f''(x) = \frac{6(x^3 - 3x)}{(x^2 + 1)^3} \).
   a) (4 points) Find the critical numbers of \( f \).
   b) (8 points) Determine where \( f \) is increasing, where \( f \) is decreasing, and find the local maxima and minima of \( f \).
   c) (4 points) Find the asymptotes of \( f \).

   \( f(x) = \frac{x^2 + 3x + 1}{x^2 + 1} \quad f'(x) = \frac{3(1 - x^2)}{(x^2 + 1)^2} \quad f''(x) = \frac{6(x^3 - 3x)}{(x^2 + 1)^3} \).
d) (5 points) Find the inflection points of \( f \) and determine where \( f \) is concave upwards and downwards.
e) (8 points) Draw a rough sketch of the graph of \( f \).

5. (20 points) Find the dimension of the right circular cylinder of greatest volume that can be inscribed in a given right circular cone of height 1 meter and radius 1 meter.

6. a) (10 points) Show that the equation \( \ln x - x + 2 = 0 \) has at least two solutions.
b) (10 points) Show that it has exactly two solutions.

7. (20 points) Find the linear approximation to \( f(x) = \frac{1}{(1 + x)^4} \) at \( a = 0 \), and use it to approximate \( \frac{1}{1.1^4} \).

8. (15 points) A cube is increasing in volume at a rate of 10cm³/sec. Find the rate of change of the surface area of the cube when one edge has length 2cm.

9. Let
\[
f(x) = \begin{cases} 
  x^2 \cot x & x \neq 0; \\
  0 & x = 0.
\end{cases}
\]
a) (10 points) Show that \( f \) is continuous at \( x = 0 \).
b) (10 points) Show that \( f \) differentiable at \( x = 0 \).