MATH 125, FINAL EXAM (common) **May 2005**

1. (6 points each) Calculate the following limits.

a) $\lim_{x\to -3} \dfrac{x^2-9}{x^2+2x-3}$.

b) $\lim_{x\to 0} \dfrac{1-\sqrt{1-x^2}}{x^2}$.

c) $\lim_{x\to 8^-} \dfrac{|x-8|}{x-8}$.

d) $\lim_{x\to 2^+} e^{\frac{1}{2-x}}$.

2. (6 points each) Find $\frac{dy}{dx}$.

a) $y = x^2 \sin 2x$.

b) $y = \dfrac{(x-1)(x-4)}{(x-2)(x-3)}$ (You may use logarithmic differentiation).

c) $xe^{-y} + ye^{-x} = 3$.

d) $y = \displaystyle\int_3^{\sqrt{x}} \frac{\cos t}{t} dt$.

3. (7 points each) Evaluate the following integrals:

(a) $\displaystyle\int (\sqrt[3]{x} - x^3 + \frac{1}{\sqrt[3]{x}})dx$.

(b) $\displaystyle\int_0^1 \frac{5x^2}{x^3+2} dx$.

(c) $\displaystyle\int_0^{\pi} x\cos(\pi + x^2)dx$

(d) $\displaystyle\int e^{2x}\sqrt{1+e^x}dx$.

4. Consider the following function and its first and second derivative:

$$f(x) = \frac{x^2+3x+1}{x^2+1} \qquad f'(x) = \frac{3(1-x^2)}{(x^2+1)^2} \qquad f''(x) = \frac{6(x^3-3x)}{(x^2+1)^3}.$$

a) (4 points) Find the critical numbers of f.
b) (8 points) Determine where f is increasing, where f is decreasing, and find the local maxima and minima of f.
c) (4 points) Find the asymptotes of f.

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d) (5 points) Find the inflection points of f and determine where f is concave upwards and downwards.
e) (8 points) Draw a rough sketch of the graph of f.

5. (20 points) Find the dimension of the right circular cylinder of greatest volume that can be inscribed in a given right circular cone of height 1 meter and radius 1 meter.

6. a) (10 points) Show that the equation $\ln x - x + 2 = 0$ has at least two solutions.
b) (10 points) Show that it has exactly two solutions.

7. (20 points) Find the linear approximation to $f(x) = \dfrac{1}{(1+x)^4}$ at $a = 0$, and use it to approximate $\dfrac{1}{1.1^4}$.

8. (15 points) A cube is increasing in volume at a rate of $10cm^3/sec$. Find the rate of change of the surface area of the cube when one edge has length 2cm.

9. Let

$$f(x) = \begin{cases} x^2 \cot x & x \neq 0; \\ 0 & x = 0. \end{cases}$$

a) (10 points) Show that f is continuous at $x = 0$.
b) (10 points) Show that f differentiable at $x = 0$.