MATH 125 **FINAL EXAM** **May 5th, 2005**

Last Name: ________________ **First Name:**________________

SSN: ____________________ **Signature:**____________________

Circle your instructor's name:

Dragnev Geisser Kim Montgomery

INSTRUCTIONS

Answer all questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of 200 points.

Problem	Value	Score
1	24	
2	24	
3	28	
4	29	
5	20	
6	20	
7	20	
8	15	
9	20	
Total	200	

1. (6 points each) Calculate the following limits.

a) $\displaystyle\lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3}$.

b) $\displaystyle\lim_{x \to 0} \frac{1 - \sqrt{1 - x^2}}{x^2}$.

c) $\displaystyle\lim_{x \to 8^-} \frac{|x - 8|}{x - 8}$.

d) $\displaystyle\lim_{x \to 2^+} e^{\frac{1}{2-x}}$.

2. (6 points each) Find $\frac{dy}{dx}$.
a) $y = x^2 \sin 2x$.

b) $y = \dfrac{(x-1)(x-4)}{(x-2)(x-3)}$ (You may use logarithmic differentiation).

c) $xe^{-y} + ye^{-x} = 3$.

d) $y = \displaystyle\int_3^{\sqrt{x}} \frac{\cos t}{t} dt$.

3. (7 points each) Evaluate the following integrals:

(a) $\int (\sqrt[3]{x} - x^3 + \frac{1}{\sqrt[3]{x}}) dx$.

(b) $\int_0^1 \frac{5x^2}{x^3+2} dx$.

(c) $\int_0^{\pi} x \cos(\pi + x^2) dx$

(d) $\int e^{2x}\sqrt{1+e^x}\, dx$.

4. Consider the following function and its first and second derivative:

$$f(x) = \frac{x^2 + 3x + 1}{x^2 + 1} \qquad f'(x) = \frac{3(1 - x^2)}{(x^2 + 1)^2} \qquad f''(x) = \frac{6(x^3 - 3x)}{(x^2 + 1)^3}.$$

a) (4 points) Find the critical numbers of f.

b) (8 points) Determine where f is increasing, where f is decreasing, and find the local maxima and minima of f.

c) (4 points) Find the asymptotes of f.

$$f(x) = \frac{x^2 + 3x + 1}{x^2 + 1} \qquad f'(x) = \frac{3(1 - x^2)}{(x^2 + 1)^2} \qquad f''(x) = \frac{6(x^3 - 3x)}{(x^2 + 1)^3}.$$

d) (5 points) Find the inflection points of f and determine where f is concave upwards and downwards.

e) (8 points) Draw a rough sketch of the graph of f.

5. (20 points) Find the dimension of the right circular cylinder of greatest volume that can be inscribed in a given right circular cone of height 1 meter and radius 1 meter.

6. a) (10 points) Show that the equation $\ln x - x + 2 = 0$ has at least two solutions.

b) (10 points) Show that it has exactly two solutions.

7. (20 points) Find the linear approximation to $f(x) = \dfrac{1}{(1+x)^4}$ at $a = 0$, and use it to approximate $\dfrac{1}{1.1^4}$.

8. (15 points) A cube is increasing in volume at a rate of $10cm^3/sec$. Find the rate of change of the surface area of the cube when one edge has length 2cm.

9. Let

$$f(x) = \begin{cases} x^2 \cot x & x \neq 0; \\ 0 & x = 0. \end{cases}$$

a) (10 points) Show that f is continuous at $x = 0$.

b)(10 points) Show that f differentiable at $x = 0$.