Spring 2005
Math 126 Final

1. The lines $x = 0$, $x = 1$, $y = 0$ and the curve $y = xe^{x^3}$ bound a region of the xy-plane. If we rotate this region about the x-axis, what is the volume of the resulting solid?

2. (a) Find the Taylor series expansion of the function $f(x) = x^3 \cos(x^2)$ about 0.

   (b) Evaluate $f^{(75)}(0)$.

   (c) Use the series in part (a) to approximate $f(1/2)$ to within 10^{-3}.

3. Find the following indefinite integrals.

   (a) $\int \frac{x}{\sqrt{3-x^4}}\,dx$ (b) $\int \tan^3 x\,dx$

4. Find the area of the surface obtained by rotating the curve $y = 2\sqrt{x}$, $0 \le x \le 8$, about the x-axis.

5. Decide whether the following infinite series are absolutely convergent, conditionally convergent, or divergent.

   (a) $\sum_{2}^{\infty} \frac{1}{n \ln n}$ (b) $\sum_{1}^{\infty} \frac{\cos n}{n^3}$

6. Sketch the curve given by the polar equation $r = 4 - 2\sin\theta$. Find the area enclosed by the curve.

7. Determine whether the following integrals are convergent or divergent.

(a) $\int_{\pi/2}^{\infty} \frac{\arctan x}{x^2}\, dx$ (b) $\int_{0}^{\pi/2} \tan x\, dx$

8. Find $\lim_{x\to\infty} (x \ln(x+2) - x \ln x)$.

9. Does the sequence $a_n = (-1)^n \cdot \frac{\ln n}{\sqrt{n}}$ converge as $n \to \infty$? If it does, find its limit.

10. Determine the radius of converge and the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{3n+1}$.