Math 125, Final Exam (common) **Spring 2004**

1. [20] Find the limits if they exist. (You may not use L'Hospital's Rule.)

   (a) [5] $\lim_{x\to 0} \tan 2x \sin(1/x)$

   (b) [5] $\lim_{x\to\infty} \dfrac{e^{3x}+5e^{2x}}{2e^{3x}-1}$

   (c) [5] $\lim_{x\to 1} \dfrac{\sqrt{x+3}-\sqrt{2x+2}}{x-1}$

   (d) [5] $\lim_{x\to 2} \ln\left(\dfrac{1}{5}\dfrac{x-2}{x^2+x-6}\right)$

2. [20] Consider
$$f(x) = \frac{(x-1)(x-2)}{|x-2|} \quad .$$

   (a) [10] Sketch the graph of f.

   (b) [10] Describe the set of points on the real axis at which f is continuous, e.g., $x \in (7,9]$ or $7 < x \leq 9$.

3. [20] Find the derivatives of the following functions:

   (a) [5] $f(x) = \dfrac{x}{\sqrt{5-2x}}$

   (b) [5] $g(x) = \sqrt[3]{1-xe^{x^2}}$

   (c) [10] $h(x) = x^{\sin x}$

4. [15] Use implicit differentiation to find the equation of the tangent line to the curve
$$y\cos(x^2) = x\cos(y^2)$$
at the point (0,0).

5. [15] If a bacteria population starts with 400 bacteria and doubles every 5 hours, then the number, $f(t)$, of bacteria after t hours is
$$f(t) = 400 \cdot 2^{t/5} \quad .$$

   (a) [5] Find the derivative of $f(t)$.

   (b) [5] Find the value of $f'(5)$.

   (c) [5] Find a linear approximation to $f(t)$ at $t = 5$.

6. [15] Consider the polynomial $p(x) = x^3 + x + 1$.

   (a) [8] Show that $p(x)$ has a root. Clearly state the reason(s).

   (b) [7] Show that $p(x)$ can not have two or more roots. Clearly state the reason(s).

7. [10] Find horizontal asymptotes (if any) of

$$f(x) = \frac{x+1}{\sqrt{x^2+1}+3x}.$$

8. [30] For the function $f(x) = \frac{1}{4}x^4 + x^3$, fill in the required information. If none, write NONE.

   (a) intervals of increase (if any):

   (b) intervals of decrease (if any):

   (c) positions of local maxima (if any):

   (d) positions of local minima (if any):

   (e) intervals where f is concave upward (if any):

   (f) intervals where f is concave downward (if any):

   (g) positions of inflection points (if any):

   (h) Sketch the graph.

9. [15] A cardboard poster containing 512 in^2 of printed region is to have a margin of 2 in. at the top, 2 in. at the bottom, and 1 in. at each side. Determine the dimensions of a rectangular piece of cardboard with the smallest area that can be used to make such a poster.

10. [15] Compute the following integrals:

    (a) [5] $\displaystyle\int e^x \sin(e^x)\, dx$

    (b) [5] $\displaystyle\int_1^{e^2} \frac{(\ln x)^2}{x}\, dx$

    (c) [5] $\displaystyle\int \frac{2-x^2}{6x-x^3}\, dx$

11. [10] Find the derivative of

$$F(x) = \int_0^{\sin x} \frac{t^2-1}{t^2+1}\, dt \quad .$$

12. [15] Let $f(x) = \dfrac{2x-1}{x+2}$.

    (a) [3] Compute the derivative, $f'(x)$.

    (b) [6] Show that the function, f, is one-to-one.

    (c) [6] Find the inverse function, $f^{-1}(x)$.