Math 126, Spring 2004, Calculus II
FINAL EXAM

Name: (please print) _____

Student ID Number: _____

Signature: _____

Circle your Professor's Name:

Bonahon   Colwell   Crowley   Haydn   Kamienny

Circle your Lecture Time: 10:00   11:00   12:00   1:00

Instructions: Try all the problems and show all your work. Answers given with no indication of how they were obtained may receive no credit.

If you need more space, write on the back of another page and clearly mark on your problem that it is continued elsewhere.
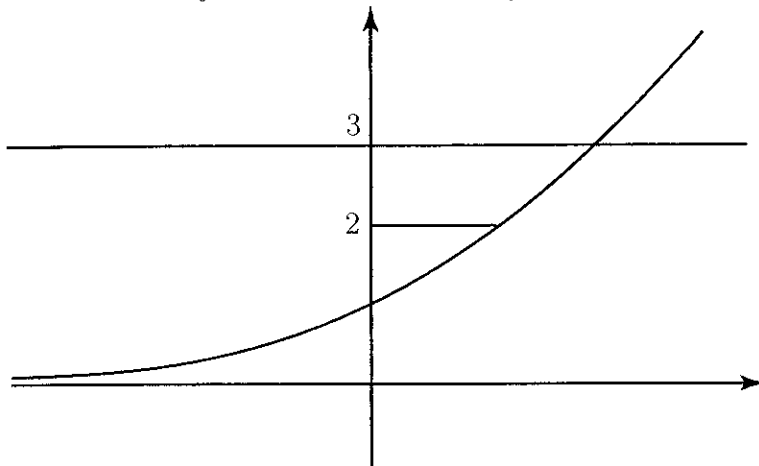
Problem	Points	Score
1	20	
2	10	
3	10	
4	20	
4c	10	
5	10	
6	10	
7	10	
8	20	
9	10	
10	20	
Total	150	

Problem 1. Find the limits:

a) (10 points) $\displaystyle \lim_{x \to 0} \frac{\int_0^x e^{-u^2}\, du}{\ln(1 - 2x)}$

b) (10 points) $\displaystyle \lim_{x \to 0} \tan x \ln x$

Problem 2. (10 points) Let $\mathcal{R}$ be the region in the (x, y)-plane bounded by $x = \ln y$, $y = 2$, and $x = 0$. Find the volume of the solid created by rotating $\mathcal{R}$ about the line $y = 3$. Make sure that you simplify your answer.

Problem 3. (10 points) A 10-foot chain weights 25 lb and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling, so that it is level with the upper end.

Before   In-between   After
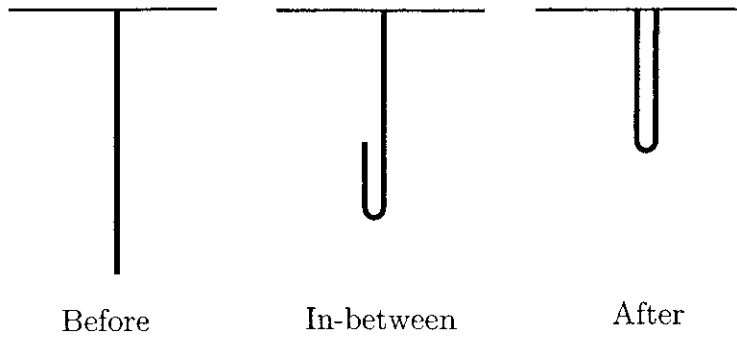
Problem 4. Compute the following integrals.

a) (10 points) $\int x \sinh x \, dx$

b) (10 points) $\int \sin^3 7x \, dx$

Problem 4 (continued).

c) (10 points) $\displaystyle\int \frac{dx}{x^4\sqrt{x^2-5}}$

Problem 5. (10 points) Determine if the improper integral $\int_0^2 \dfrac{1}{z^2 + z - 2} \, dz$ converges or diverges, and give its value if it converges.

Problem 6. (10 points) We want to approximate the value of the integral $\int_1^3 \frac{1}{x^2}\,dx$ by using the Midpoint Rule and subdividing the $[1,3]$ into n intervals of equal length. How large should we take n if we want the error to be less than 0.0002?

Problem 7. (10 points) Noting that $\dfrac{1}{(2n+1)(2n-1)} = \dfrac{1}{2}\left(\dfrac{1}{2n-1} - \dfrac{1}{2n+1}\right)$, find the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n-1)} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \frac{1}{7\cdot 9} + \dots$$

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Problem 8. Determine if the following series are absolutely convergent, conditionally convergent or divergent. Make sure that you justify your answers.

a) (10 points) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{n2^n}{(n-1)!}$

b) (10 points) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}$

Problem 9. (10 points) Write the Taylor series of $f(x) = x\sin(x^3)$ about $x = 0$. What is $f^{(22)}(0)$?

Problem 10. Give the radius of convergence and the interval of convergence of the following power series:

a) (10 points) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

b) (10 points) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(x-1)^n}{2n+1}$