

INSTRUCTIONS

Answer all questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of 200 points.

1. [24 points] In each case, evaluate the limit if it exists, including $\pm\infty$. Be sure to show your work. If no limit exists, explain why.

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 + 4x - 21}$

(b) $\lim_{x \rightarrow 0} \frac{\sin x}{|x|}$

(c) $\lim_{x \rightarrow 1} \frac{\sqrt{4x} - \sqrt{x+3}}{\sqrt{x} - 1}$

(d) $\lim_{x \rightarrow \infty} \frac{e^{3x} - e^{4x}}{e^{7x}}$

2. [24 points] Find the derivatives of the following functions.

(a) $f(x) = (x^2 + x - 1) \sin x^2$.

(b) $f(x) = \frac{x^2}{x+3}$.

(c) $f(x) = \ln(1 + e^{x^3})$

(d) $f(x) = \int_0^{x^2} \frac{dt}{t^4 + 1}$.

3. [30 points] Evaluate the following integrals.

(a) $\int_0^1 (3x - 1)^4 dx$

(b) $\int_0^3 |x^2 - 1| dx$

(c) $\int \frac{\cos x}{(1 + \sin x)^2} dx$

(d) $\int e^{2x} \sqrt{1 + e^x} dx$

(e) $\int \frac{1}{\sqrt{x}(\sqrt{x} + 1)} dx$

4. [16 points] (i) Give the definition of: f is differentiable at a .

(ii) Use the definition to show that the function

$$g(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable at 0.

5. [16 points] Find the equation of the tangent line at the point $P(2, 3)$ to the curve given by the equation

$$xy^3 - x^2y = 5x^3 + 2.$$

6. [16 points] Consider the equation

$$\cos x + \sin x = 2 - 3x.$$

(i) Show that the equation has at least one real root.

(ii) Show that the equation has at most one real root.

7. [22 points] For the function $f(x) = x^3 - 6x^2 + 9x - 4$ fill in the required information and sketch the graph. If none, write NONE.

(i) Horizontal asymptotes (if any):

(ii) Vertical asymptotes (if any):

(iii) Positions of local maxima (if any):

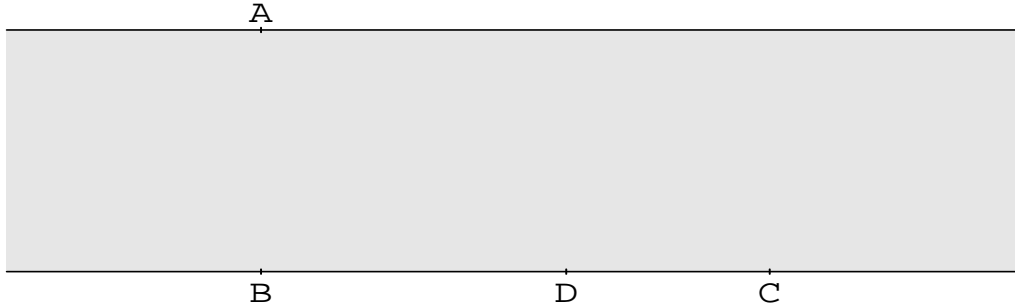
(iv) Positions of local minima (if any):

(v) Positions of inflection points (if any):

(vi) Intervals where f is concave upward (if any):

(vii) Intervals where f is concave downward (if any):

8. [20 points] A lake has straight parallel shores one mile apart. A man is in a row boat at a point A on one shore directly opposite the point B on the opposite shore. He wants to get to a point C which is two miles from B along the opposite shore. He will row the boat to some point D between B and C and then jog the remaining distance from D to C. If he rows at a speed of 3 miles per hour and jogs at a speed of 6 miles per hour, how should he choose the position of D so as to minimize the time it takes him to get from A to C?



9. [16 points] Find $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \frac{i^{1/2}}{n^{1/2}}$ and explain your answer.

10. [16 points] Find the domain of the function $f(x) = \sqrt{5 - e^{3x}}$. Find a formula for the inverse function f^{-1} , and find the domain of f^{-1} .