Math 125, Spring 2002, Calculus I

FINAL EXAM

Problem 1. Evaluate the following limits (finite or infinite). Use only the techniques seen in this course. In particular, you are not allowed to use L'Hospital's rule if you know what this is.

a. (8 points) $\lim_{x \to 4^+} \sqrt{\dfrac{4x}{x-4}}$

b. (8 points) $\lim_{x \to 0^+} \dfrac{\ln x}{4 + 3\ln x}$

Problem 2. Evaluate the following limits (finite or infinite). Use only the techniques seen in this course. In particular, you are not allowed to use L'Hospital's rule if you know what this is.

a. (8 points) $\lim_{x \to \infty} (\sqrt{e^{2x} + e^x} - e^x)$

b. (8 points) $\lim_{x \to e} \dfrac{\ln x - 1}{x - e}$ (Possible hint: Could this be a derivative?)

Problem 3. Differentiate the following functions:

a. (8 points) $f(x) = \ln(1 + x^x)$

b. (8 points) $g(x) = \dfrac{(x^2+1)^5}{\sqrt{e^x+1}(5\sin(x)+1)^2}$ (Suggestion: logarithmic differentiation, by computing the derivative of the natural log of the function)

Problem 4. (8 points) Differentiate the following function and simplify your answer:

$$h(x) = \int_{\sqrt{x}}^{\sqrt{\ln x}} e^{t^2}\, dt$$

Problem 5. (8 points) Evaluate the following integral: $\displaystyle\int_0^2 |1 - x^2|\, dx$

Problem 6. Evaluate the following integrals and simplify your answer:

a. (8 points) $\displaystyle\int_2^4 \frac{1}{x \ln x}\, dx$

b. (8 points) $\displaystyle\int_{-1}^1 \frac{x^2}{\sqrt{x^3+9}}\, dx$

Problem 7. (19 points) Consider the function $f(x) = (1 - 2x - x^2)e^{-x}$. Its first and second derivatives are $f'(x) = (x^2 - 3)e^{-x}$ and $f''(x) = (-x^2 + 2x + 3)e^{-x}$. You do not need to check these computations, and YOU DO NOT NEED TO GRAPH THE FUNCTION $f(x)$ either. Indicate, by writing the answer on the dotted lines below:

(1) the intervals where the function is increasing (if any): ....

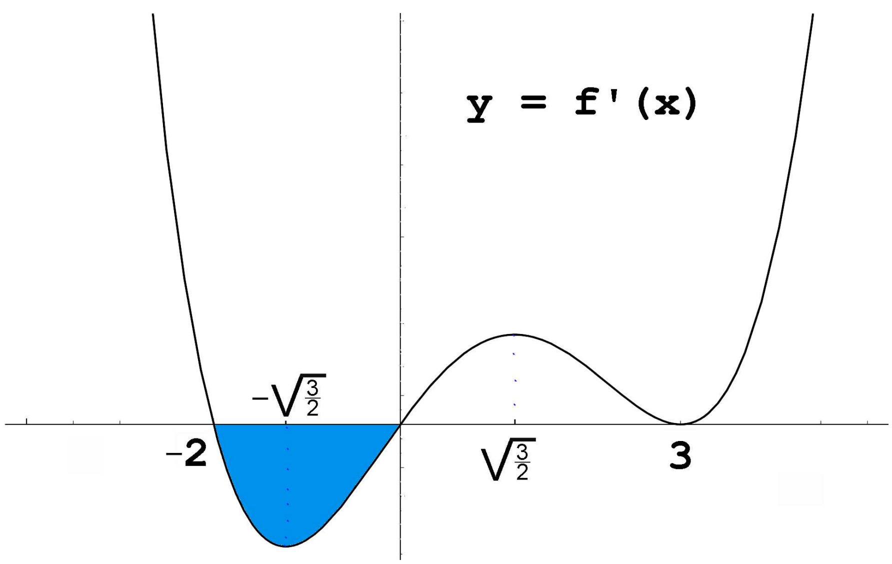
(2) the x-coordinates of local maxima (if any): ....

(3) the x-coordinate of local minima (if any): ....

(4) the intervals where the function is concave up (if any): ....

(5) the x-coordinates of inflection points (if any): ....

Problem 8. (15 points) The graph of the derivative $f'(x)$ of a function $f(x)$ is given below.



Indicate by writing the answer on the dotted lines below:

(1) the x-coordinate(s) of all local maxima of the function $f(x)$: ....

(2) the x-coordinate(s) of all local minima of the function $f(x)$: ....

(3) the x-coordinate(s) of all inflection points of the function $f(x)$: ....

(4) specify the interval(s) where the function $f(x)$ is concave down: ....

(5) given that $f(0) = 1$ and that the shaded region has area $\dfrac{108}{5}$, the value of $f(-2)$ is equal to: ....

Problem 9. (8 points) Find the slope of the tangent line to the curve $(x-2y)^3 = 2y^2x-3$ at the point $(1,1)$.

Problem 10. (10 points) Write the following limit as an integral, and compute it.

$$\lim_{n\to\infty} \frac{1}{n}\left[\sqrt{\frac{4n+3}{n}} + \sqrt{\frac{4n+6}{n}} + \sqrt{\frac{4n+9}{n}} + \ldots + \sqrt{\frac{4n+3(n-1)}{n}} + \sqrt{\frac{4n+3n}{n}}\right]$$

Problem 11. (8 points) Knowing that $\sqrt[5]{32} = 2$ (since $2^5 = 32$), use linear approximation to give an approximate value for $\sqrt[5]{33}$ with 4 digits after the decimal point.

Problem 12. (15 points) A rock is dropped into a lake and an expanding circular ripple results. When the radius of the ripple is 8 feet, the radius is increasing at the rate of 3 feet/second. At what rate is the area enclosed by the ripple changing at this time?

Problem 13. (15 points) An ant colony grows in such a way that, t days after we began observing it, the number of ants increases at the rate of $4t^2 + 30$ individuals per day. If it started with 1000 ants, how many will there be after 30 days?

Problem 14. (15 points) Somewhere in the Middle West, two roads go very straight and meet in the city of Troyville. The first road runs South-North, while the second road runs East-West. A truck is moving South on the first road at the constant speed of 60 miles per hour and, at noon, is just 100 miles north of Troyville. A car drives East on the second road at the speed of 80 miles per hour and, at noon, is exactly 50 miles East of Troyville. When is the car closest to the truck?

Problem 15. (15 points) Bacteria grow in a Petri dish (namely a dish containing a solution of nutrients), in such a way that the area of the dish covered by the bacteria increases exponentially. If it takes 1 day for the bacteria to cover $\frac{1}{3}$ of the dish, and 2 days to cover $\frac{1}{2}$ of it, when will the dish be completely full?