MATH 125 -- FINAL EXAM -- SPRING 2001

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1. (30 points) Differentiate the following functions:

   a) $(1 + e^{x^2})^5$ b) $\dfrac{x}{1+\sqrt{x}}$ c) $\displaystyle\int_1^{x^3} \frac{sin(t)}{1+t^3}dt$

2. (30 points) (a) Give the definition of: $f'(a)$ is the derivative of f at a.

   (b) State the Mean Value Theorem (including its hypothesis).

3. (30 points) Find:

   (a) $\displaystyle\int_0^{\sqrt{\pi}} x sin(x^2)dx$ (b) $\displaystyle\int_{ln2}^{ln3} \frac{e^x}{1+e^x}dx$ (c) $\displaystyle\int x^2\sqrt{1+x}\,dx$

4. (20 points) Consider the function $f(x) = \begin{cases} \dfrac{|x^2 - 9|}{x - 3} & if \quad x \neq 3 \\ 6 & if \quad x = 3 \end{cases}$ defined for all $x \in \boldsymbol{R}$.

   Describe all the $x \in \boldsymbol{R}$ at which f is continuous. *<u>You must justify your answer</u>*.

5. (40 points) Let $g(x) = x^9e^x$. Complete each statement. Work out the answers below. The work itself and reasons for the answers must appear to get any credit.

   i) g is increasing on the intervals ______________________________

   ii) g is decreasing on the intervals ______________________________

   iii) g has local maxima at ______________________________

   iv) g has local minima at ______________________________

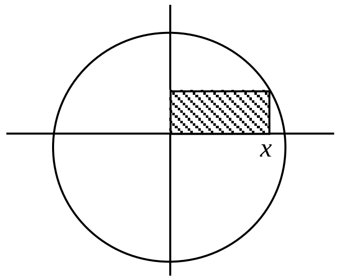
   v) Which answers if any in iii) and iv) are absolute maxima or minima? __________

   vi) The graph of g is concave up on the intervals ____________________

   vii) The graph of g is concave down on the intervals ____________________

   viii) The graph of g has points of inflection where x = ____________________

6. (25 points) A rectangle with sides on the positive x-axis and positive y-axis is inscribed in the circle of radius 1 with center at the origin. If $A(x)$ is the area of this rectangle for $3/5 \le x \le 4/5$, so A:[3/5,4/5]$\to R$, find those $x \in$ [3/5,4/5] where A attains a maximum value and where A attains a minimum value, or say no such values exist. The rectangle is pictured just below.



7. (25 points) If $N \geq 2$ is a positive integer, use Riemann sums to show that

$$(1/2) + (1/3) + \cdots + (1/N) \leq \ln N.$$