Math 126 - FINAL - Spring 2001

S. Bohacek, F. Bonahon, C. Haskell, F. Lin, R. Sacker, and E. Verona (in alphabetical order)

1. (15 Points) Find the volume of the solid obtained by rotating about the x axis the region in the first quadrant bounded by $x = y^2$, $y = -x + 2$ and $y = 0$.

2. (15 Points) A cable that weighs 2 lb/ft is used to lift 500 lb of coal up a mineshaft 300 ft deep. Find the work done in lifting the coal with the cable.

3. (15 Points) Evaluate $\displaystyle\int x\cos(x)dx$.

4. (10 Points) Evaluate $\displaystyle\lim_{x\to 0} x\ln(x)$.

5. (15 Points) Evaluate $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$.

6. (20 Points) Evaluate $\displaystyle\int \frac{x^2+2}{(x^2+x+1)(x-1)}\,dx$.

7. (20 Points) A reservoir has a dam at one end. The dam is in the shape of a trapezoid that is 100 m wide at the top, 40 m wide at the bottom and 30 m high. Assume the water level is at the top of the dam. Find the force of the water acting on the dam.

   (Note: The density of water is 1,000 kg/m^3 and the acceleration due to gravity is 9.8 m/s^2.)

8. (15 Points) Sketch the curve whose equation in polar coordinates is $r = 1 + \sin\theta$. Then determine the area of the region consisting of those points which are inside of this curve and below the x axis.

9. (10 Points) Consider the curve whose equation in polar coordinates is $r = \cos\theta + 2\sin\theta$. Find an equation for this curve in Cartesian coordinates, namely in the traditional (x, y)–coordinates. An implicit equation for y in terms of x is acceptable.

10. (20 Points) Determine convergence or divergence for the following two series. Please state any test you use and verify that all the necessary conditions for applying that test are satisfied.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{2n+1}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(\ln\ n)^2}{(\ln\ 5)^n}$

11. (15 Points) Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n4^n}$.

    (a) Find the radius of convergence.

    (b) Find the interval of convergence, i.e., the set of all x for which the series converges.

12. (15 Points) Expand $\displaystyle\frac{1}{\sqrt[5]{1+x}}$ as a power series about $a = 0$. Show at least the first four nonzero terms. *Hint:* Use the Binomial Theorem.

13. (15 Points) The n-th Taylor polynomial for e^x is $T_n(x) = 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \ldots + \dfrac{x^n}{n!}$. Find the smallest n so that for every $x \in [0, 1]$ the n-th Taylor polynomial approximates e^x to within an error of at most 0.1. Show why your choice of n works.