Math 218 Final Examination: Spring 2010

Name (Printed)_____ **Student ID**_____

Signature (handwritten)_____

Directions. **Do not OPEN the test booklet until instructed to do so.** Fill out your name, signature, and student ID number on the lines above and check the boxes below to indicate your lecture and discussion times.

You may use a calculator and *one* 8-1/2 by 11-inch sheet of *handwritten* notes (both sides may be written on). No books or other notes are permitted.

When an answer box is provided, copy your answer into that box. Numerical answers should be evaluated to be either decimals or fractions in lowest terms. **Numerical answers alone are not sufficient; you MUST indicate how you derived them (show your work).** When submitting a numerical answer which is a decimal, use the number of decimal places warranted by the data. Be sure to include units when appropriate.

Some problems are worth more points than others; the value of a problem is indicated in parentheses following the problem number. The exam totals 200 points.

After you are instructed to open the test, make sure that all 12 pages are present (not including the normal tables and t-tables that are distributed separately). **Note that the separately distributed sheet of tables is printed on both sides.**

Please check your lecture time and professor:

☐ 9:00 MWF (Lin)	☐ 9:00 MWF (Haskell)	☐ 10:00 MWF (Lin)
☐ 10:00 MWF (Haskell)	☐ 11:00 MWF (Lin)	☐ 11:00 MWF (Lytvak)
☐ 12:00 MWF (Polunchenko)	☐ 12:00 MWF (Kim/Voineagu)	☐ 1:00 MWF (Voineagu)
☐ 1:00 MWF (Rosen)	☐ 2:00 MW (Voineagu)	☐ 2:00 MW (Piterbarg)

Please check your TTh discussion time:

☐ 8:00 ☐ 9:00 ☐ 10:00 ☐ 11:00 ☐ 12:00 ☐ 1:00 ☐ 2:00 ☐ 3:00 ☐ 4:00

Do not write on this page below this line!

1 (20 pts)		7 (20 pts)		
2 (26 pts)		8 (20 pts)		
3 (20 pts)		9 (6 pts)		Total Points
4 (20 pts)		10 (16 pts)		
5 (20 pts)		11 (18 pts)		
6 (14 pts)				
(120 pts)		(80 pts)		

Problem 1. (20 points) A financial services company offers two investment opportunities, Alpha and Bravo. With probability 0.6, Alpha yields an annual rate of return of 3%, otherwise its return is 6%. Bravo returns either 5% or 10%; when Alpha returns 3%, the probability that Bravo returns 5% is 0.7 and when Alpha returns 6%, the probability that Bravo returns 5% is 0.8. *Hint: you may find a probability tree helpful in this problem.*

a) Find the probability that Bravo yields a return of 10%.

a)

b) Given that Bravo yields a return of 10%, find the probability that Alpha yields a return of 6%.

b)

c) A client invests \$1000 in Alpha and \$2000 in Bravo. Find the expected gain in the client's total investment after one year.

c)

Problem 2. (26 points) A supermarket has two express lanes. Let X denote the number of customers in the first lane and Y the number of customers in the second lane at 5pm on Monday afternoons. The joint probability distribution of X and Y is shown below.

		Y			
		0	1	2	3
X	0	0.25	0.1	0	0
	1	0.2	0.1	0.05	0
	2	0	0.1	0.1	0
	3	0	0	0.05	0.05

a) Which lane generally has more customers at 5pm on Monday afternoons? Circle your answer and explain.
   (i) First lane has more customers.
   (ii) Second lane has more customers.
   (iii) They have equal numbers of customers.

b) Find the conditional distribution of Y given $X = 1$. Put a box around your answer.

c) Find $\mathrm{Cov}(X, Y)$.

c)

d) Find the correlation ρ.

d)

e) Find the distribution of the total number of customers in the two express lanes combined at 5pm on Monday afternoons. Put a box around your answer.

Problem 3. (20 points) A financial services company offers investors a selection of 30 stocks; 20 are NYSE stocks, and the remaining are NASDAQ stocks.

a) A client selects a portfolio of 12 stocks at random *without* replacement.

(i) Find the probability that exactly 5 NYSE stocks are selected.

a)(i)

(ii) Find the expected value and standard deviation of the number of NYSE stocks selected.

Expected value:

Standard deviation:

b) A client selects a portfolio of 12 stocks at random *with* replacement.

(i) Find the probability that exactly 5 NYSE stocks are selected.

b)(i)

(ii) Find the expected value and standard deviation of the number of NYSE stocks selected.

Expected value:

Standard deviation:

Problem 4. (20 points) Under ordinary circumstances, people blink their eyes at random points in time at an average rate of 2 blinks every 5 seconds. Research has found that a person being deceitful blinks more frequently. The FBI is interested in developing a lie detector test based on this research.

a) Find an integer N such that, under ordinary circumstances, the chance that a person blinks their eyes more than N times in a 5-second period is less than 10%.

a)

b) Compute the probability that, under ordinary circumstances, less than 1 second elapses between successive eye blinks.

b)

Problem 5. (20 points) A state licensing exam that is given annually has been designed so that the scores are normally distributed with a mean of 68% and a standard deviation of 15%.

a) What percentage of the test-takers score between 65% and 89%?

a)

b) If 45% of the test-takers pass the test and become licensed, what is the lowest passing score?

b)

c) If 650 people take the test one year, what is the probability that more than 300 of them pass?

c)

Problem 6. (14 points) The time it takes security at Stochastic Airport to process one person is exponentially distributed with a mean of 30 seconds. Gary is late for his plane and runs up to go through security. There are 35 people in front of him. What is the probability that Gary and all of the people in front of him will get through security in less than 20 minutes? *Hint: use the Central Limit Theorem.*

Answer:

Problem 7. (20 points) A recent survey of 800 American adults found that 570 supported the President's new health care legislation.

a) Assuming that the sampling was random and independent, determine a 90% confidence interval for the true proportion p of American adults who support the new law.

a)

b) What is the smallest sample size that is needed in order to be *guaranteed* a margin of error no greater than $\pm 3\%$ at the 95% level of confidence?

b)

Problem 8. (20 points) The number of full-time employees at 6 randomly selected office furniture dealers in Los Angeles is:

50 78 41 33 35 33

a) Find a 90% confidence interval for the mean number of full-time employees at all office furniture dealers in Los Angeles.

Interval:

b) What assumption about the population is required for the interval to be valid?

Problem 9. (6 points) Company records indicate that drivers get an average of 65,000 miles on the company's All-Weather radial tires. Hoping to improve that figure, the company has added a new polymer to the rubber that it hopes will protect the tires from deterioration caused by extreme temperatures and humidity. To test whether the new polymer is effective, the company will perform a hypothesis test and test drive 15 cars equipped with the new tires until the tires wear out. Formulate the null and alternative hypotheses of the test. Carefully explain the meaning of any parameters that appear in your null and alternative hypotheses.

H_0 :

H_a :

Meaning of the parameter(s):

Problem 10. (16 points) Commercial fishermen are sometimes hindered in their efforts by the presence of dolphins. Fortunately, 40% of the time that dolphins are sighted near fishing boats, they leave of their own accord, probably just to get away from the noise of the boat. It has been suggested that if the fishermen transmitted the sounds of a killer whale under water, the dolphins might be more apt to leave. A test is done to determine if this procedure is effective. The null and alternative hypotheses of the test are:

$$H_0 : p = .4$$
$$H_a : p > .4$$

On 52 occasions when dolphins were sighted near fishing boats, the sounds of a killer whale were transmitted under water. The dolphins left on 24 of these occasions.

a) To what does the p in the null and alternative hypothesis refer? Circle the best answer.

   i) The probability that dolphins, sighted near fishing boats, leave of their own accord.

   ii) The probability that dolphins, sighted near fishing boats, leave when sounds of a killer whale are transmitted under water.

   iii) The p-value of the test.

   iv) The proportion of times in the sample that dolphins leave.

b) Calculate the p-value.

b)

c) What does the p-value tell us? Circle the best answer.

   i) It is the probability the alternative hypothesis is true.

   ii) It is the probability, calculated assuming that transmitting sounds of a killer whale has no effect, that on 52 occasions when dolphins are sighted near fishing boats they will leave on at least 24 of those occasions.

   iii) It is the probability that dolphins, sighted near fishing boats, leave of their own accord.

   iv) It is the probability that dolphins, sighted near fishing boats, leave when sounds of a killer whale are transmitted under water.

Problem 11. (18 points) A company that produces cell phones claims its standard phone battery lasts at least 35 hours on average. A consumer advocacy group doubts this is true and conducts a hypothesis test. The null and alternative hypotheses of the test are:

$$H_0 : \mu \geq 35 \text{ hours}$$
$$H_a : \mu < 35 \text{ hours}$$

where μ is the actual average lifetime of the company's batteries. They measure the lifetimes (in hours) of 7 randomly selected batteries. The data collected are:

35 32 30 31 38 34 31

Assume that the lifetimes of the batteries are normally distributed.

a) Choose a test statistic and find its numerical value for the data collected.

Formula or expression for the test statistic:

Value of the test statistic:

b) Formulate the rejection rule and write it in the box below. Based on that decide whether the company claim can be rejected at the 5% significance level. Circle your answer.

Reject company claim. Don't reject company claim.

Rejection Rule:

c) At which of the following levels of significance can the company claim be rejected? Circle all that apply. Show your work.

20% 12% 4% 2.5% 0.5%