

Name (printed) \_\_\_\_\_ Student ID \_\_\_\_\_

Signature (handwritten) \_\_\_\_\_

**Instructions**

- This exam consists of 13 pages including this cover page. Before starting the exam, check that you have all 13 pages. Please notice that the separately distributed sheet of tables is printed on both sides.
- There are ten problems and the point distribution among the problems is shown below. There are 200 points total. Answer all the problems. Please notice that the 4th and 10th problems take up *two* pages each.
- You may use a calculator as long as it is not capable of remote communication (eg no cell phones). You may refer to both sides of an 8.5 x 11 inch sheet of handwritten notes. No books or other notes are permitted.
- Numerical answers alone are not sufficient. **You must indicate how you derived your answers (show work) to obtain full credit.** Points may be deducted if you do not justify your final answer. Please indicate clearly if you continue your work on the back of the page.
- When an answer box is provided, put your final answer in the box.
- When submitting a numerical answer that is a decimal, use the number of decimal places warranted by the data.
- When submitting an answer that is a fraction, reduce it to lowest terms.
- Be sure to include units in your answers where appropriate.
- If you cannot do part a) of some problem, but you need the answer for b), then you can get partial credit for showing you know what to do. You could write, “let p be the answer to a),” and solve b) in terms of p.

**Please check your lecture time and professor:**

- |   |   |  |
|---|---|--|
| <input type="checkbox"/> 9:00 MWF (Yin)       | <input type="checkbox"/> 9:00 MWF (Haskell) | <input type="checkbox"/> 10:00 MWF (Haskell) |
| <input type="checkbox"/> 10:00 MWF (Lin)      | <input type="checkbox"/> 11:00 MWF (Lin)    | <input type="checkbox"/> 11:00 MWF (Yin)     |
| <input type="checkbox"/> 12:00 MWF (Voineagu) | <input type="checkbox"/> 12:00 MWF (Lin)    | <input type="checkbox"/> 1:00 MWF (Voineagu) |
| <input type="checkbox"/> 1:00 MWF (Lytvak)    | <input type="checkbox"/> 2:00 MW (Dumett)   | <input type="checkbox"/> 2:00 MW (Lytvak)    |

*Do not write on this page below this line!*

1 (20 pts)		6 (16 pts)	
2 (20 pts)		7 (20 pts)	
3 (20 pts)		8 (20 pts)	
4 (22 pts)		9 (20 pts)	
5 (20 pts)		10 (22 pts)	
(100 pts)		(100 pts)	

Total Points
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*Problem 1.* A large credit card company finds that 70% of new cardholders pay their first monthly bill in full. Moreover, 60% of those that pay their first bill in full also pay their second bill in full. On the other hand, only 10% of those who pay less than the full amount the first month, pay their second bill in full.

- a) Suppose 2 new cardholders are chosen at random. What is the probability that they both pay their first monthly bill in full? (The number of new cardholders is so large that you need not worry about whether the choice is made with replacement or without.)

a)

- b) Find the probability that a randomly chosen customer pays both her first and second bills in full.

b)

- c) Find the probability that a randomly chosen customer pays exactly one of her first two bills in full.

c)

- d) If a randomly chosen customer paid her second bill in full, what is the probability that she also paid her first bill in full?

d)

*Problem 2.* Ann invests \$100 in AAA Corp. stock and \$100 in PDQ Corp. stock. One year later the values of her two investments are likely to have changed. Let  $X$  denote the value of her investment in AAA Corp.'s stock and  $Y$  denote the value of her investment in PDQ Corp.'s stock at this point in time. Both  $X$  and  $Y$  are measured in dollars. The probability distributions of  $X$  and  $Y$  are shown below.

$x$	90	100	110	120
$P(X = x)$	0.1	0.3	0.4	0.2

$y$	100	110	120	130
$P(Y = y)$	0.2	0.4	0.3	0.1

a) Find the expected value of  $X$ .

a)

b) Find the standard deviation of  $X$ .

b)

c) Assuming  $X$  and  $Y$  are independent, find the probability that one year later her total investments will be worth more than \$230.

c)

*Problem 3.* Let  $X$  denote the proportion of a new drug that has been assimilated by the body 1 hour after ingestion. Notice that the value of  $X$  varies from person to person. The probability density function of  $X$  is given by

$$f(x) = 3(1 - x)^2, \quad 0 \leq x \leq 1.$$

Notice that the expected value of  $X$  is 0.25 (you do not need to show this).

- a) When a person takes the drug, what is the probability that more than 50% of the drug is assimilated 1 hour after ingestion?

a)

- b) Find the standard deviation of  $X$ .

b)

- c) When 60 people take the drug, what is the probability that after one hour they assimilate, on average, at least 20% of the drug? If you are unable to find the standard deviation in part b) and you need it for part c), use the value of 0.22.

c)

*Problem 4.* Joe has a pushcart on the beach from which he sells flip-flops and sunglasses. The table below shows the joint distribution of the number of pairs of flip-flops  $X$  and the number of pairs of sunglasses  $Y$  that he sells each day.

X	Y					
	0	1	2	3	4	
0	.1	.1	.1	0	0	.3
1	.1	.1	.1	0	0	.3
2	0	0	0	.2	0	.2
3	0	0	0	.1	.1	.2
	.2	.2	.2	.3	.1	

a) Are  $X$  and  $Y$  independent? Circle yes or no and explain.

**YES**

**NO**

b) Find  $\text{Cov}(X, Y)$ .

b)

c) Find the conditional distribution of  $X$  given that Joe sells 3 pairs of sunglasses. Put a box around your answer.

*Problem 4. continued*

- d) Joe makes a profit of \$10 on each pair of flip-flops he sells and a profit of \$15 on each pair of sunglasses he sells. He has fixed expenses of \$3 a day for a business license. Find the expected value and standard deviation of his daily net income from selling flip-flops and sunglasses.

Expected value:

Standard deviation:

*Problem 5.* Mei, a Math 218 student, crosses Stochastic Street twice every day (including weekends); once in the morning and once in the evening. Each time she crosses the street she has to wait for the green walk light. The amount of time she waits is uniformly distributed between 0 and 60 seconds.

- a) Find the probability that tomorrow morning she will have to wait more than 40 seconds for the green walk light.

a)

- b) Find the probability that next week (May 11 - May 17 inclusive) she will have to wait more than 40 seconds exactly 3 times. Remember that she crosses the street twice each day.

b)

- c) Find the probability that she will wait more than 40 seconds at least 27 times during the month of June. (Please note that there are 30 days in June and remember that she crosses the street twice each day.) If you use an approximation, show that it is valid to do so.

c)

*Problem 6.* John receives a lot of SPAM emails claiming that he's won the lottery. These emails arrive according to a Poisson process with, on average, 3 emails arriving every hour.

- a) One day, 25 minutes elapse between the first and second of these emails. What is the probability that more than 30 minutes elapse between the second and third emails?

a)

- b) What is the probability that fewer than two of these emails arrive in the next 30 minutes?

b)



*Problem 7.* According to an international study, the height of American males is normally distributed with a mean of 176 cm and a standard deviation of 7 cm.

- a) Find the probability that the height of a randomly selected American male is between 167 cm and 174 cm.

a)

- b) Assume now that the height of Canadian males is also normally distributed with the same mean of 176 cm but an unknown standard deviation. If only 5% of Canadian males are taller than 190 cm, find the standard deviation of the height of Canadian males.

b)

- c) A Math 218 discussion section has 18 students. Four of these students are males whose height is between 167 and 174 cm. Three students in the discussion section are chosen at random to do a presentation at the board. What is the probability that at least two of those that are chosen are males whose height is between 167 and 174 cm?

c)

*Problem 8.* The research division of an insurance company wishes to estimate the mean value of the personal property owned by residents of a particular zip code. The values of the personal property of 4 randomly selected residents of this zip code are:

250    265    270    290

where the values are given in thousands of dollars. Assume that the values of the personal property owned by all residents of this zip code are normally distributed.

- a) Construct a 95% confidence interval for the mean value of the personal property owned by residents of this zip code.

a)

- b) A previous study suggested that the standard deviation of the value of the personal property owned by all residents of this zip code is 20 thousand dollars. Based on this new piece of information, construct a 95% confidence interval for the mean value of the personal property owned by residents of this zip code.

b)

- c) The insurance company wishes to estimate the mean to within plus or minus 2 thousand dollars at the 95% confidence level. How large of a sample do they need to obtain such an estimate? (You may assume that the standard deviation of the value of the personal property owned by residents of this zip code is 20 thousand dollars as in part b).)

c)

*Problem 9.* Data gathered by physical education coaches internationally show that, on average, basketball players can jump 50 cm high. A new medicine has been developed that purportedly makes basketball players jump higher. In a study to determine if this new medicine is effective, 16 basketball players will take the medicine for 2 weeks and the heights that they can jump at the end of the 2 weeks will be measured.

- a) Formulate the null and alternative hypotheses of the study.

$H_0$ :
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$H_a$ :
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- b) Choose a test statistic and find the rejection region if the study is to be conducted at the 5% significance level.

Test statistic:
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Rejection Region:
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- c) At the end of the 2 weeks it is found that the 16 players in the sample can jump on average 60 cm and the sample standard deviation is 15 cm. What should the researchers conclude? State the conclusion in standard English that people without knowledge of statistics can understand and put a box around your answer.

- d) Which of the following correctly estimates the p-value? Circle your answer.

- i) less than 0.01
- ii) between 0.01 and 0.05
- iii) between 0.05 and 0.1
- iv) greater than 0.1

*Problem 10.* A year ago 70% of all consumers preferred to buy pre-washed lettuce in bags (as opposed to a head of unwashed lettuce). To investigate if this is still the case, an agricultural organization conducts an investigation. The null and alternative hypotheses of the test are:

$$H_0 : p = 0.7$$

$$H_a : p \neq 0.7$$

In a random sample of 100 consumers they find that 62 of them prefer to buy pre-washed lettuce in bags.

a) To what does the  $p$  in the null and alternative hypothesis refer? Circle the best answer. You do not need to show your work, but if you do, you may receive partial credit even if your answer is incorrect.

- i) It is the  $p$ -value of the test.
- ii) It is the proportion of consumers in the sample that prefer to buy pre-washed lettuce in bags.
- iii) It is the proportion of all consumers one year ago that preferred to buy pre-washed lettuce in bags.
- iv) It is the proportion of all consumers today that prefer to buy pre-washed lettuce in bags.

b) Calculate the  $p$ -value.

b)

c) What does the  $p$ -value tell us? Circle the best answer. You do not need to show your work, but if you do, you may receive partial credit even if your answer is incorrect.

- i) It is the proportion of all consumers that prefer to buy pre-washed lettuce in bags.
- ii) It is the probability, calculated assuming the null hypothesis is true, that in a random sample of 100 consumers, at most 62% or at least 78% prefer to buy pre-washed lettuce in bags.
- iii) It is the probability that the null hypothesis is correct.
- iv) It is the probability that the null hypothesis should be rejected.

**This problem is continued on the following page.**

*Problem 10. continued*

- d) A consumer group performs a similar hypothesis test with the same null and alternative hypotheses but uses a sample of 200 consumers. A total of 124 consumers in their sample prefer to buy pre-washed lettuce in bags. Which of the following statements is correct? Circle the best answer. You do not need to show your work, but if you do, you may receive partial credit even if your answer is incorrect.
- i) The agricultural organization has stronger evidence against  $H_0$ .
  - ii) The agricultural organization and the consumer group have equally strong evidence against  $H_0$ .
  - iii) The consumer group has stronger evidence against  $H_0$ .
  - iv) We cannot tell who has the stronger evidence without more information.
- e) Based on the consumer group's sample, find a 90% confidence interval for the proportion of consumers that prefer to buy pre-washed lettuce in bags.

e)