MATH 218 FINAL EXAM
MAY 4, 2006

Problem 1 (25 pts). *A portfolio manager offers two assets to potential investors. Half of the investors select Asset A; the remaining investors select Asset B. Asset A has three possible returns: 8%, with a probability of 1/4; 12%, with a probability of 1/2; 16%, with a probability of 1/4. Asset B also has three possible returns: 8%, with a probability of 1/3; 14%, with a probability of 1/3; 20%, with a probability of 1/3.*

(a) *Draw the appropriate probability tree. Be sure to include the labels of events, probabilities, and conditional probabilities.*
(b) *Find the probability that a randomly chosen investor earns a return of at least 10%.*
(c) *Given that a randomly chosen investor earns a return of at least 10%, find the probability that the investor selected Asset A.*
(d) *Given that a randomly chosen investor earns a return of at least 10%, find the probability that the return is greater than 15%.*

Problem 2 (20 pts). *The annual rate of return (in percent),* X*, of stock A has the following probability distribution:*

x	-10	0	10	20	30
$P(X = x)$	0.17	0.20	0.26	0.20	0.17

For example, -10 *represents a loss of 10% of the initial investment, while* 10 *represents a gain of* 10% *of the initial investment. After one year, an investor will hold the amount of the initial investment plus any amount gained and minus any amount lost.*

(a) *Find the expected annual rate of return on stock A (in percent).*
(b) *Find the standard deviation of the annual rate of return on stock A (in percent).*
(c) *Jane starts with $1,000, borrows $200 at a constant annual risk-free rate of* 6%*, and then invests the sum of* $\$1,200$ *in stock A. After one year, Jane's investment will be worth*

$$\$1200(1 + 0.01X) - \$200(1.06) = \$988 + \$12X \quad .$$

Find the expected value of Jane's investment (in dollars and cents) after one year.
(d) *Find the standard deviation of Jane's investment (in dollars and cents) after one year.*

Problem 3 (20 pts). *The two common types of errors made by programmers are syntax errors and logic errors. For a simple language such as BASIC the number of such errors is usually small. Let* X *denote the number of logic errors and* Y *denote the number of syntax errors made on the first run of a BASIC program. The joint probability distribution of* X *and* Y*,* $P_{XY}(x, y)$*, is given by:*

Y

		0	1	2	3	4	
	0	0.08	0.01	0.07	0.11	0.03	
X	1	0.04	0.08	0.16	0.08	0.04	
	2	0.03	0.06	0.12	0.06	0.03	

(a) *Find Cov* (X, Y).

(b) *Are the random variables* X *and* Y *independent? Justify your answer.*

(c) *Given that the number of syntax errors on the first run is greater than two, find the probability that the number of logic errors on the same run is two.*

(d) *Find* $P(X + Y = 4)$.

Problem 4 (20 pts). *FlyByNight Airlines has noticed that* 4% *of the people that make reservations on a flight do not show up for the flight, so they book* 50 *passengers on a plane that has only* 48 *seats.*

(a) *What is the probability that there will be a seat available for every person who shows up for the flight?*

(b) *What is a probability that* at least one *of the passengers will not get a seat.*

(c) *What are the expected value and standard deviation of the number of passengers that show up?*

(d) *One day all* 50 *passengers who bought tickets show up for the flight and* 47 *of them turn out to be participants in the FlyByNight Frequent Flier program. Two passengers out of* 50 *are selected randomly and bumped off the flight. What is the probability that* both *of those bumped off are* not *members of the Frequent Flier program?*

Problem 5 (20 pts). *In an effort to improve its operating system, Microsoft puts together a team to find all the errors in the* 40 *million lines of code that comprise its operating system. Suppose the following:*

- *Errors occur at random locations in the code with, on average,* $1/10$*th of an error occurring in every* 10,000 *lines of code.*
- *The number of errors that occur in any given section of code is independent of the number that occur in any non-overlapping section of code.*
- *Every day the team searches through* 10,000 *lines of code and finds all the errors in that section of code.*

(a) *Let* X *be the number of errors that are found in a* 5*-day period. What kind of probability distribution does* X *have? Give a formula for the probability that exactly* k *errors are found in a* 5*-day period.*

(b) *How many errors would you expect the team to find in a* 5*-day period?*

(c) *What is the expected number of days that elapse between consecutive errors being found?*

(d) *Find the probability that less than* 7 *days elapse between the first and second errors being found.*

(e) *The team sets to work and finds no errors during the first* 10 *days. What is the probability that they find the first error on the* 15*th day of work?*

Problem 6 (20 pts). *While today we all share the prospect of a long and happy life, this was not always the case. For example, scientists estimate that the mortality rate for Cro-Magnon man approximately* 25,000 *years ago followed the following graph:*

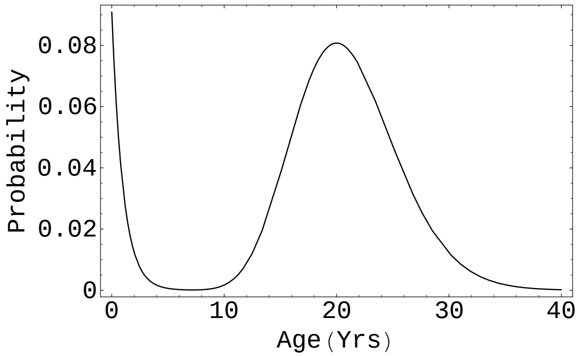

FIGURE 1. Mortality rates for Cro-Magnon. The area under the curve between two ages represents the proportion of Cro-Magnons who died between those ages.

Only 0.03% *of Cro-Magnons lived beyond the age of* 40, *so we haven't bothered to plot that part of the probability distribution.*

Analytically, Figure 1 is the graph of

$$y = f(x) = \frac{1}{1.1}\left(0.1e^{-x} + \frac{x^{20}}{20!}e^{-x}\right).$$

You may use the following fact: if n *is a non-negative integer, then*

$$\int_0^\infty x^n e^{-x}\,dx = n!$$

The questions to be answered appear on the next page.

(a) *Show that* $f(x)$ really is *a probability density function on* $x \geq 0$.
(b) *Estimate, from the graph, at approximately what ages mortality is highest for Cro-Magnons.*
(c) *Compute the mean lifetime of a Cro-Magnon.*

Problem 7 (20 pts). *An airline has noticed that the weights of passengers (along with their luggage) on a small commuter flight from Burbank to Fresno are normally distributed with a mean of* 185 *pounds and a standard deviation of* 35 *pounds.*

(a) *What proportion of the passengers weigh (along with their luggage) more than* 180 *pounds?*
(b) *The airplane used on this flight seats* 42 *passengers and has a weight limit of* 8000 *pounds. What is the probability that, on a sold-out flight, the total weight of the* 42 *passengers and their luggage will exceed the weight limit?*
(c) *To reduce the chance of exceeding the weight limit the airline wants to impose a weight limit for each passenger. What weight limit should they impose if they only want* 10% *of the passengers to be affected by its imposition?*

Problem 8 (15 pts). *A highway safety researcher is designing a highway sign. After some research she finds that the maximum distances at which signs like hers can be read are normally distributed with a mean that depends on the sign and a standard deviation of* 42 *feet. To estimate the mean maximum distance at which her sign can be read she selects* 5 *drivers at random and finds that the maximum distances (in feet) at which they can read her sign are:*

575 490 510 425 545

(a) *Estimate, with* 95% *confidence, the mean maximum distance at which drivers can read her sign.*
b) *How much larger of a sample does she need if she wishes to estimate the mean maximum distance to within plus or minus* 5 *feet?*

Problem 9 (20 pts). *In the United States it is expected that* 19.2 *million or* 8% *of adults age* 18 *or older will suffer from clinical depression within the next year. It has been suggested that the stress of college increases the risk of clinical* *depression. In a random college survey 39 out of 357 students fit the criteria for clinical depression. A hypothesis test is performed to determine whether college students are more susceptible to clinical depression.*

(a) *Formulate the null and alternative hypotheses.*
(b) *State the test statistic that should be used for this test.*
(c) *Determine the point estimate for the population proportion.*
(d) *Draw a picture of the rejection rule at the* $\alpha = 5\%$ *level.*
(e) *Compute the* p*-value of the data.*
(e) *For which of the following levels of significance should the null hypothesis be rejected? Circle all that apply.*

10%   5%   2.5%   1%   0.5%

Problem 10 (20 pts)**.** *The accompanying data are the times (in seconds) that it took a crew of workers to assemble a toy truck at Tyko Industries' assembly plant, when their efforts were sampled at random:*

192   172   184   200   179

(a) *At the* 10% *significance level, can we conclude that the mean assembly time for this truck is not* 3 *minutes?*
(b) *Estimate the* p*-value for the data.*