MATH 218 FINAL EXAM

May 3, 2005

Problem 1. Your company receives widgets from 3 suppliers, A, B, and C. Sixty percent of the widgets come from A, thirty percent come from B, and the rest come from C. Company A is excellent, and only 5 percent of their widgets are defective. Company B is not so good, and 20 percent of their widgets are defective. Company C is pretty bad, and half of their widgets are defective. You take a random widget arriving at your company, and test it. Write D for the event that it is defective.

1a) Draw a tree diagram to represent this situation. Include all events and probabilities involved — individual, conditional, and joint (intersection) probabilities. [REMINDERS: branches from the root are labeled with individual probabilities, further branches are labeled with conditional probabilities, and terminal nodes represent intersections and should be labeled with the (joint) probability of that intersection].

1b) What is the chance that the tested widget is defective?

1c) Suppose the tested widget *is* defective. What is the chance that it came from A?

1d) Suppose the tested widget *is* defective. What is the chance that it came from B?

1e) Suppose the tested widget *is* defective. What is the chance that it came from C?

1f) Suppose the tested widget *is NOT* defective. What is the chance that it came from A?

1g) [Yes or no.] Should the probabilities for 1c), 1d), and 1e) add up to 1?

1h) [Yes or no.] Should the probabilities for 1c), 1f) add up to 1?

Problem 2. A supermarket has two express lines. Let X denote the number of customers in the first express line and Y denote the number of customers in the second express line at any given time. During non-rush hours, the joint probability distribution of X and Y is given by the following table:

	Y \ X	0	1	2	3	
Y	0	0.1	0.2	0	0	
Y	1	0.2	0.25	0.05	0	
Y	2	0	0.05	0.05	0.025	
Y	3	0	0	0.025	0.05	

(a) Fill the marginal distribution of X in the table.

(b) Compute the expected value of the random variable X.

(c) Compute the variance of the random variable X.

(d) What is the probability that at any given time during non-rush hours the total number of customers in the two express lines is at least 4?

(e) Are the random variables X and Y independent? Justify your answer.

Problem 3. 20% of `BadDay`-brand tires are defective.

(a) Find the probability that, among nine randomly selected tires of this brand, **at most** two are defective.

(b) Find approximately the probability that, among 160 randomly selected tires of this brand, **at least** 40 are defective.

(c) A tire store has 15 `BadDay`-brand tires in stock, of which three are defective. You buy five tires of this brand. What is the probability that you bought **exactly** two defective tires? Write your answer in the form

$$\frac{\binom{a}{b}\binom{c}{d}}{\binom{x}{y}}$$

for suitable numbers a, b, c, d, x, y.

(d) A government inspector visits a `BadDay` factory and checks, one by one, five tires coming off the assembly line. What is the probability that the fifth tire will be the only defective tire found by the inspector?

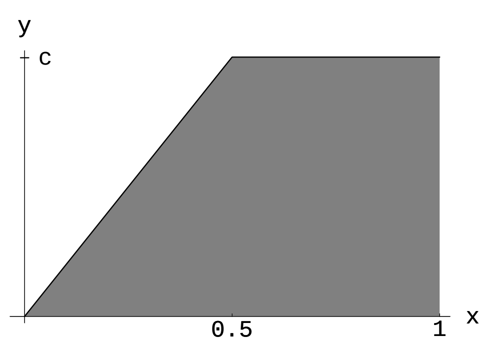
Problem 4. Patients arrive at a 24-hour medical facility according to a Poisson process. On average, there is 1 arrival per hour. The interarrival times are independent and exponentially distributed.

(a) Find the probability that there is at least one arrival in the first two hours.

(b) Find the probability that the time between the first and second arrivals is less than one-half hour.

(c) The records for the week begin at 8:00 a.m. on Monday. Find an approximate probability that the 81-st arrival after 8:00 a.m. occurs within 72 hours (i.e., before 8:00 a.m. on Thursday).

Problem 5. Let X be a random variable with the probability density function $f(x)$. The graph of the probability density function $f(x)$ is shown below and $f(x)$ is given by



$$f(x) = \begin{cases} 2cx, & \text{if } 0 \le x \le 0.5; \\ c, & \text{if } 0.5 \le x \le 1; \\ 0, & \text{otherwise.} \end{cases}$$

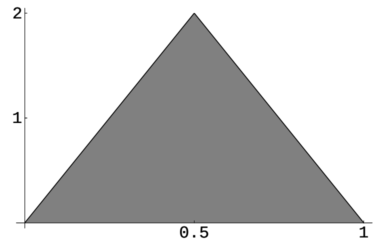
(a) What is the value of c, the height of the graph?

(b) Find the mean μ of the random variable X.

(c) Find that value of w such that $P(X \le w) = 0.95$.

(d) $P(0 < X < \frac{1}{4}) = \frac{1}{12}$. Find $P(X > \frac{3}{4} \mid X > \frac{1}{4})$.

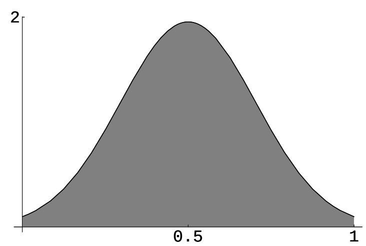
Problem 6. A random variable X has a probability density function as given below:



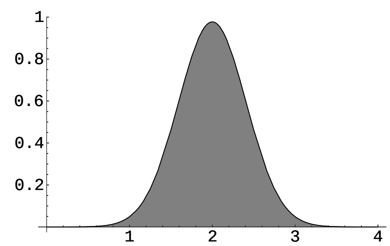
Four independent random variables X_1, X_2, X_3, X_4 with the same distribution are added, to yield

$$Y = X_1 + X_2 + X_3 + X_4.$$

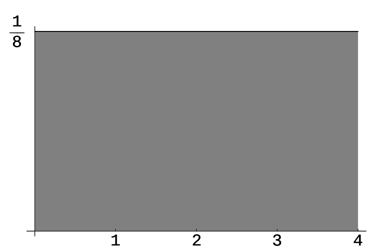
(A) Which of the following graphs most closely resembles the probability density function $f_Y(y)$ for the random variable Y? Circle one.



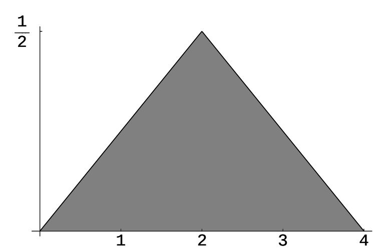
(a)



(b)



(c)



(d)

(B) Circle all the statements that are true.

(1) The graph in (c) is the graph of a probability density function.

(2) $E(Y) = E(X_1) + E(X_2) + E(X_3) + E(X_4)$.

(3) $E(Y) = 2E(X_1)$.

(4) $Var(Y) = Var(X_1) + Var(X_2) + Var(X_3) + Var(X_4)$.

(5) $f_Y(y) = 0$ for all $y > 4$.

(6) The maximum value of $f_Y(y)$ is achieved when $y = 0.5$.

(7) $f_Y(y) = 0$ for all $y > 1$.

Problem 7. A market researcher for a consumer electronics company wants to study the television viewing habits of residents of a particular city. A random sample of 41 respondents is selected, and each respondent is instructed to keep a detailed record of all television viewing in a particular week. For this sample the viewing time per week has a mean of 15.3 hours and a sample standard deviation of 3.8 hours. Assume that the amount of time of television viewing per week is normally distributed.

(a) Construct a 95% confidence interval for the mean amount of television watched per week in this city.

(b) If the market researcher wants to make another survey in a different city, what sample size is required to be 95% confident of estimating the population mean to within ± 2 hours assuming that the population standard deviation is known to be 5 hours?

Problem 8. The owner of a downtown parking lot suspects that the average time cars use the parking lot is actually greater than the one reported by the person she hired to run the lot. The receipts as provided by the employee indicate that, on average, each car is parked for 3.5 hours. To determine whether the time parked is actually greater than the 3.5 hours reported by the employee, the owner observed 141 cars and noticed that the time spent on the lot by those cars was on average 3.73 hours with a sample standard deviation of 1 hour. Assume that the amount of time cars are parked is normally distributed.

(a) Formulate the null and the alternative hypotheses.

(b) Determine the test statistic. Evaluate it numerically.

(c) Estimate the p-value. Circle the correct range for the p-value.

$$< .5\% \qquad [.5\%, 1\%) \qquad [1\%, 2.5\%) \qquad [2.5\%, 5\%) \qquad \geq 5\%$$

(d) Can the owner conclude at the 1% significance level that the time parked is actually greater than 3.5 hours?

(e) Suppose that the standard deviation of the time cars are parked in the parking lot is known and is $\sigma = 1$ hour. If the real mean time cars are parked in the parking lot is 3.6 hours, determine the probability of a type II error at a 1% significance level.

Problem 9. A pharmaceutical company has developed a drug that has provided relief for 2 out of 3 patients using the drug. After making some modifications, the company wishes to determine whether the proportion of patients that are helped by the drug has changed in any way. The company decides to test this by using 200 randomly selected patients with a level of significance of 5%.

(a) Formulate appropriate null and alternative hypotheses.

(b) State the rejection rule.

(c) In response to a survey, 150 patients reply that the drug has helped them. Compute the value of the test statistic.

(d) Should the null hypothesis be rejected at the 5% level of significance? Explain.

(e) Find the p-value.