## MATH 218 FINAL EXAMINATION <br> May 4, 2004

Name (Print): $\qquad$ Student ID:
Signature (handwritten): $\qquad$
Directions: Fill out your name, signature, and student ID number on the lines above then check the lecture time. Do not open the test booklet until instructed to do so. After you are so instructed, make sure all 13 pages are present (not including the two tables, which are distributed separately). There are not supposed to be any blank pages.

On this examination, you may use a calculator and one $8-1 / 2$ by 11-inch sheet of handwritten notes (both sides may be written on). No books or other notes are permitted.

Use an approximation and the continuity correction where appropriate. When an answer box is provided, you are required to copy your answer there. Numerical answers should be evaluated to be either decimals or fractions in lowest terms. Numerical answers alone are not sufficient; you MUST indicate how you derived them (show your work).

## Check one lecture time:

| $\square$ | 2:00 MW Dumett |
| :--- | :--- | :--- |
| 12:00 MWF Jaffrey |  |
| 10:00 MWF Styrkas |  |$\quad \square$| 10:00 MWF Haskell |
| :--- |
| 11:00 MWF Kukavica |$\quad$| 12:00 MWF Haydn |
| :--- |
| 12:00 MWF Vorel |

DO NOT WRITE ON THIS PAGE BELOW THIS LINE

| $1(20 \mathrm{pts})$ |  | $6(20 \mathrm{pts})$ |  |
| :--- | :--- | :--- | :--- |
| $2(20 \mathrm{pts})$ |  | $7(20 \mathrm{pts})$ |  |
| $3(25 \mathrm{pts})$ |  | $8(20 \mathrm{pts})$ |  |
| $4(20 \mathrm{pts})$ |  | $10(20 \mathrm{pts})$ |  |
| $5(20 \mathrm{pts})$ |  | $\mathrm{T}(100 \mathrm{pts})$ |  |
| $\mathrm{T}(100 \mathrm{pts})$ |  |  |  |

Problem 1. Forty members of the Healthy and Happy Hiker's club are going together on a day hike. The club provides the members with one sandwich and one beverage for lunch. They can choose a beef, turkey or vegetarian sandwich and a soda, ice tea or carrot juice to drink. The choices the members made are shown in the table below.

|  | Sandwich <br> Beverage |  |  |
| :--- | :---: | :---: | :---: |
| beef | turkey | vegetarian | Total |
| soda <br> ice tea <br> carrot juice | 6 | 2 | 8 |
| Total | 10 | 12 | 40 |

a) When a member is selected at random, the events \{ice tea\} and $\{$ turkey $\}$ are independent, the events $\{$ carrot juice\} and \{beef\} are mutually exclusive, and $P($ carrot juice|vegetarian $)=0.75$. Use this information to fill in the gaps in the table.

Answer:
b) What is the probability that a randomly selected member chose to drink ice tea?

Answer:
c) What is the conditional probability that a randomly selected member chose a vegetarian sandwich given that they chose to drink ice tea?

Problem 2. In a certain company, $40 \%$ of all the employees are female. Eighty percent of the female and ninety percent of the male employees are union members.
a) Organize the data in a tree or contingency table indicating all the relevant probabilities.
b) Find the percentage of union members that are female.

Problem 3. Statistics show that $25 \%$ of all CDs sold at Tower Records are classical music CDs and $30 \%$ are pop music.
a) Yesterday, during the morning hours only 12 CDs were sold. What is the probability that exactly 3 were pop music CDs?

## Answer:

b) If 100 CDs were sold yesterday, what is the probability that at least 31 were classical music CDs?

Answer:
c) Jane goes to Tower Records to buy some CDs. She looks around and finds 5 pop, 6 jazz, and 4 classical music CDs that she likes. Unfortunately she can not afford them all, so she picks 4 of them at random. What is the probability that she buys exactly 2 jazz CDs?

Problem 4. The number of users at the local automatic banking machine follows a Poisson distribution. The mean number of users is 15 per hour.
a) Find the probability that there will be at least 2 users in the next 10 minutes.

Answer:
b) What is the average interval of time between two consecutive users?

Answer:
c) A user just left and 2 minutes elapsed without any user. What is the probability that at least an additional 5 minutes will elapse before the next user will arrive?

Problem 5. Dr. Jay Z. a psychologist, has established that the attention span $X$ in hours of a kindergarten child in class is distributed according to $f(x)=a x^{2}(1-x)$ for $0 \leq x \leq 1$ and $f(x)=0$ otherwise.
a) Find the value of $a$.

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Answer:
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b) Find the expected attention span of a kindergarten child.

Answer:
c) A kindergarten child was selected at random. Her attention span was found to be greater than 30 minutes. Find the probability that it is less than 45 minutes.

Answer:

Problem 6. Random variables $X$ and $Y$ are distributed according to

|  |  | $Y$ |  |
| :---: | :---: | :---: | :---: |
|  |  | 0 | 1 |
|  | 0 | 0.1 | 0.1 |
| $X$ | 1 | 0.1 | 0.2 |
|  | 2 | 0.4 | 0.1 |

a) Compute $\operatorname{Cov}(X, Y)$.

Answer:
b) Find $P(Y=1 \mid X \leq 1)$
c) Find $P(X+Y>1)$.

Problem 7. The manager at a car dealership noticed that the number $X$ of cars sold by each salesman per week follows the following distribution

| $a$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $P(X=a)$ | 0.1 | 0.3 | 0.25 | 0.2 | 0.15 |

a) Find the expected value and the standard deviation of $X$.
expected value:
standard deviation:
b) Find the probability that a salesman will sell more than 110 cars in a year. (One year has 52 weeks.)

Problem 8. The lifetime of a toaster from the company Toaster's Choice has a normal distribution with standard deviation 1.5 years. A random sample of 400 toasters was drawn yielding the sample lifetime average of 6 years.
a) Find a $90 \%$ confidence interval for the mean lifetime of the toasters.

Answer:
b) What sample size is needed to find the mean lifetime of the toasters to within plus or minus 0.05 years at the same $90 \%$ confidence level?

Problem 9. The "Vegetable and Pumpkin Patch" nursery claims that at least $85 \%$ of the seeds they sell will germinate. Cleophilia, one of their customers, has had little success in her gardening ventures. She decided to check on the nursery's claim.
a) Formulate appropriate null hypothesis $H_{0}$ and alternative hypothesis $H_{1}$, and enter them in the boxes:

| $H_{0}:$ |
| :--- |
| $H_{1}:$ |

b) She took 200 seeds and found that 160 of them germinated. Find the $P$-value.

Answer:
c) Should she reject the nursery's claim at the $5 \%$ significance level?

Problem 10. Before 1995, The Dow Jones Industrials (DJI), a key economic indicator, returned on average $12 \%$ annual growth earnings. Sally, a 2000 economic student, suspects that the economy has fundamentally changed during the five year period 1995-1999.
a) Formulate the null and alternative hypothesis.

| $H_{0}:$ |
| :--- |
| $H_{1}:$ |

b) Choose an appropriate test statistic and formulate the rejection rule at $10 \%$ level of significance.

Rejection rule:
c) For the period 1995-1999, the yearly returns for the DJI were:

| 1995 | 1996 | 1997 | 1998 | 1999 |
| :--- | :--- | :--- | :--- | :--- |
| $33 \%$ | $26 \%$ | $23 \%$ | $16 \%$ | $25 \%$ |

Which conclusion should she reach?

Reject $H_{0}$ (yes or no)?:

