Spring 2003 Math 218 Final Examination

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Problem 1. A firm that assembles personal computers buys its chips from two suppliers, S1 and S2. After testing the chips, they find that 7% of the chips supplied are defective. Moreover, they find that 45% of the defective chips and 40% of the effective chips are supplied by S1. The remainder is supplied by S2. We would like to know separately the percentage of S1's chips that are defective and the percentage of S2's chips that are defective. Let A denote the event that a randomly chosen chip is defective and B the event that a randomly chosen chip is supplied by S1.

a) Organize the data with a tree diagram and indicate **all** relevant probabilities.

b) Are A and B independent events? Numerically justify your answer.

c) What is the percentage of defective chips among those supplied by S1? What is the percentage of defective chips among those supplied by S2?

d) Two defective chips are randomly chosen. Find the probability that both are supplied by S1.

Problem 2. A vice president of a large biotechnology company is interviewing a candidate for director of research (which depends on probability and statistics) and asks the following questions.

a) The DNA molecule contains four types of bases indicated by the letters A, C, G, and T. In how many ways can three letters be selected with replacement from A, C, G, and T, including order?

b) In how many ways can the four letters A, C, G, and T be ranked (e.g., in order of importance)?

c) If three letters are selected at random with replacement from the letters G and T, in how many ways can exactly two G's be chosen without regarding order?

d) If three letters are chosen at random without replacement from A, C, G, and T, find the probability that the letter G will be chosen.

e) If the letters A, C, G and T occur with probabilities 0.31, 0.31, 0.25, and 0.13, respectively, and are assumed to be independent, find the probability of "ACT" occurring in that order.

Problem 3. The Fancy Cookie Factory makes chocolate, pecan, and fruit cookies. Visitors are offered trays of cookies to taste. Each tray contains 3 chocolate, 5 pecan, and 4 fruit cookies.

a) If a visitor picks at random 4 cookies from a full tray, what is the probability that he picks 2 or 3 fruit cookies?

b) Assume that each of 15 visitors takes a full tray and picks at random 4 cookies to taste. Find, in decimal form, the probability that exactly 6 of the 15 visitors pick 2 or 3 fruit cookies.

c) Twenty-five percent of the cookies produced at the Fancy Cookie Factory are chocolate cookies. A restaurant orders 350 cookies without specifying the type. Because of that, the cookies are selected at random, packed, and shipped. What is the approximate probability that between 250 and 275 cookies (inclusive) in this shipment are pecan or fruit cookies?

Problem 4. Imperfections in an operating system cause a computer server to crash according to a Poisson distribution with an average frequency of 2 times per week.

a) Find the probability that there will be at least one crash in a 4-day period.

b) The name of the distribution of the time interval T between two consecutive crashes is $\boxed{}$ with expected value $\boxed{}$ and standard deviation $\boxed{}$.

c) Find the probability that the next crash will occur within one day from the previous crash.

d) Suppose that the server has been working for 6 days without a crash. Find the probability that the next crash will occur during the 7^{th} day.

Problem 5. Given the following probability density function for the amount of time X required to fill customer orders (elapsed time from order entry to receipt by customer in weeks): $f(x) = \frac{1}{2}(2 - x)$, for $0 < x < 2$ weeks and 0 otherwise.

a) Find $E(X)$.

b) Find $\mathrm{Var}(X)$.

c) Calculate $P(\frac{1}{2} < X < 1\frac{1}{2})$.

d) If the cost of inquiries about orders or complaints requiring further action is given by the relationship Cost $= 10X + 2$, what is the expected cost you would anticipate given the above distribution of X?

Problem 6. Weights of boxes of 16 fish sticks have a normal distribution with mean 16.2 ounces, and standard deviation 1 ounce.

a) What is the probability that a box weighs less than 16 ounces?

b) Ninety-five percent of all boxes weigh more than c ounces. Find c.

c) A government inspector will take a random sample of 4 boxes and fine the company if the average weight in this sample is under the weight claimed on the boxes. If the company is willing to run the risk of being fined 1% of the time, with what weight should it label its boxes?

Problem 7. The number of times a student visits the University bookstore in a one-month period is a random variable X with the following probability distribution:

x	0	1	2	3
$P(X = x)$	0.05	0.25	0.50	0.20

Assume that the students visit the bookstore independently of one another.

a) Find the average number of times one student visits the bookstore in a one-month period.

b) Find the standard deviation of the number of times one student visits the bookstore in a one-month period.

c) Find the exact probability that students Peter, Paul and Mary will visit the bookstore a total of at least eight times in a one-month period.

d) Find the approximate probability that 100 students will visit the bookstore a total of 180 times or fewer in a one-month period.

Problem 8. Assume the assets of the community banks are normally distributed with a standard deviation σ of 161 million dollars. The American Bankers Association surveyed 110 banks and found their total assets to be 24,200 million dollars.

a) Find the point estimate of μ, the mean assets for all community banks.

b) Find a 95% confidence interval for μ.

c) Would the length of a 90% confidence interval be larger or smaller? Explain.

d) What sample size is needed to determine μ to within ± 10 million dollars with 95% confidence?

Problem 9. The reported average price for a gallon of self-serve regular unleaded gasoline is no more than \$1.75 in the United States; you believe that the figure is higher. Your random survey of 6 stations produces the following prices.

\$1.88 \$2.05 \$1.68 \$1.81 \$1.95 \$1.77

Assume gasoline prices are normally distributed.

a) Formulate the null and alternative hypotheses.

b) Choose an appropriate test statistic and find a rejection rule at the 1% level of significance.

c) Determine whether your data provide enough evidence to reject the null hypothesis at the 1% level of significance.

d) At which of the following levels of significance can the null hypothesis be rejected? Circle all that apply:

0.15 0.10 0.05 0.025 0.01 0.005

Problem 10. In a sample of 144 drivers, 78 preferred car A to car B, all factors considered.

a) Would you reject $H_0 : p \leq 0.5$ where p is the proportion of the population preferring car A to car B, in favor of $H_a : p > 0.5$, if $\alpha = 0.05$ is used for the level of significance?

b) Find the P-value.

c) Find the 95% confidence interval for p.