MATH 218 FINAL EXAMINATION
May 1, 2002

Name (Print): _____ Student ID: _____

Signature (handwritten): _____

Directions: Fill out your name, signature, and student ID number on the lines above then check the lecture time. **Do not open the test booklet until instructed to do so.** After you are so instructed, **make sure all 13 pages are present** (not including the two tables, which are distributed separately). There are not supposed to be any blank pages.

On this examination, you may use a calculator and one 8-1/2 by 11-inch sheet of *handwritten* notes (both sides may be written on). No books or other notes are permitted.

Use the continuity correction where appropriate. When an answer box is provided, you are required to copy your answer there. Numerical answers should be evaluated to be either decimals or canceled fractions. **Numerical answers alone are not sufficient; you MUST indicate how you derived them (show your work).**

Check one lecture time:

☐ 10:00 MWF Goukasian ☐ 11:00 MWF Goukasian ☐ 10:00 MWF Kukavica
☐ 11:00 MWF Kukavica ☐ 11:00 MWF Haydn ☐ 12:00 MWF Vorel
☐ 2:00 MW Haskell ☐ 2:00 MW Piterbarg ☐ 12:00 MWF Lin
☐ 9:00 MWF Hu

———————————DO NOT WRITE ON THIS PAGE BELOW THIS LINE ———————————

1 (20 pts)		6 (20 pts)	
2 (15 pts)		7 (20 pts)	
3 (25 pts)		8 (20 pts)	
4 (20 pts)		9 (20 pts)	
5 (20 pts)		10 (20 pts)	
T (100 pts)		T (100 pts)	

Problem 1. (20 pts)A group chosen for a medical study contains 60 participants with 12 smokers among them. There are a total of 36 male participants, and 9 of them smoke.

a) Organize the data using the following contingency table.

	Male	Female	*Total*
Smoker			
Non-smoker			
Total			

b) What is the probability that a randomly selected participant is a smoker?

Answer for (b):

c) What is the probability that a randomly selected participant is either female or a non-smoker?

Answer for (c):

d) Are the events "is male" and "is a smoker" independent? Explain.

Answer for (d):

e) A **male** participant is chosen at random. What is the probability that he is a smoker?

Answer for (e):

Problem 2. (15 pts)A violin maker gets boards of wood from two suppliers: "Westwood Lumber" and "Musical Timber". Westwood Lumber supplies 40% of the total number of boards and Musical Timber supplies the rest. Of all the boards coming from Westwood Lumber, 1% are flawed, and 2% of those supplied by Musical Timber are flawed. If a board is chosen at random and is found to be flawed, find the probability that it came from Westwood Lumber.

Answer:

Problem 3. (25 pts) A company is forming a division to launch a new product. There are 20 employees who are qualified to join this division. Of these 20 employees, 15 were trained in operations management and the remainder were trained in marketing.

a) If 5 employees are assigned at random to work in this division, find the probability that exactly 4 of them were trained in operations management.

Answer for (a):

b) Suppose that 4 of the 5 employees assigned to work in the new division *were* trained in operations management. Each week for the next 32 weeks, one of the 5 employees will be selected at random to lead a division meeting. Find the (exact) probability that 30 of the meetings will be led by an employee trained in operations management.

Answer for (b):

c) Suppose that 4 of the 5 employees assigned to work in the new division were trained in operations management. Each week for the next 32 weeks, one of these employees will be selected at random to lead a division meeting. Estimate the probability that between 18 and 30 of the meetings (inclusive) will be led by an employee trained in operations management.

Answer for (c):

Problem 4. (20 pts)A classical study by F. Thorndike examined the number of misconnected calls from pay telephones in a large transportation terminal. According to the study, the number of misconnected calls has a Poisson distribution with a rate of 1.75 per minute.
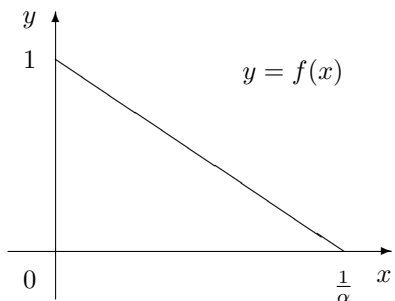
    a) Find the probability that at least two misconnected calls occur in a 2-minute period.

Answer for (a):

    b) Find the probability that the time between two consecutive misconnected calls is longer than 1.5 minutes.

Answer for (b):

Problem 5. (20 pts) A biologist finds that the lifetime X (in months) of a fruit fly has the probability distribution $f(x) = 1 - \alpha x$ for $0 \leq x \leq \frac{1}{\alpha}$ (and $f(x) = 0$ otherwise).

a) Find the value of α.

Answer for (a):

b) Find the expected lifetime $E(X)$ of a fruit fly.

Answer for (b):

c) What is the probability that a fruit fly will live longer than 0.5 months?

Answer for (c):

Problem 6. (20 pts) The machine that fills beer bottles at Happy Hops Brewing Co. can be adjusted to different fill weights to account for different sizes of bottles. However, the machine is not precise and the actual fill weights of the beer bottles are normally distributed with mean, μ, equal to the setting chosen by the user and standard deviation, σ, equal to 0.3 oz.

    a) Suppose the machine is set to fill bottles to a fill weight of 20.25 oz. What proportion of the bottles will have fill weights of less than 20 oz?

Answer for (a):

    b) The manager of the company wants to ensure that at least 95% of 20 oz. bottles actually do contain at least 20 oz. of beer. At what fill weight should they set the machine?

Answer for (b):

Problem 7. (20 pts) Let X be the number of cavities that develop in a 6-month period in the mouth of a child that uses the new brand of toothpaste "Cavifree". The distribution of X is shown below.

c	0	1	2	3
$P(X = c)$	0.3	0.4	0.2	0.1

a) A family has three children and they all use Cavifree. Assuming that the number of cavities acquired by any one child is independent of the number acquired by any other child, find the probability that between them they acquire at most one cavity in a 6-month period.

Answer for (a):

b) Find the expected value and the standard deviation of X.

Expected value:

Standard deviation:

c) A boarding school has 150 students and they all use Cavifree. What is the probability that the students acquire more than a total of 200 cavities in a 6-month period. (Again, you may assume that the number of cavities acquired by the different students are independent.)

Answer for (c):

Problem 8. (20 pts)A financial economist wants to estimate the proportion of Chief Executive Officers (CEOs) over 65 years of age. Suppose this economist has access to a list of all CEOs. If this list is used as a basis for the study, she can randomly select a sample of CEOs, contact them, and ascertain their ages. From 144 of the CEOs selected, she learns that 108 are more than 65 years of age.

a) Find a point estimate for the proportion of CEOs over 65 years of age.

Answer for (a):

b) Construct a 90% confidence interval to estimate the proportion of CEOs over 65 years of age.

Answer for (b):

c) How many more CEOs should the economist take to estimate the proportion of CEOs over 65 years of age with 90% confidence to within ±0.03?

Answer for (c):

Problem 9. (20 pts)A weight-loss company "Sleek and Slender" claims that the average weight loss of its customers is at least 25 pounds. After a bad experience with this company, Fred, an unsatisfied customer, wants to perform sampling in order to reject the claim of Sleek and Slender and possibly sue them. He decides to take 4 randomly chosen customers and to use a level of significance of 5%. He has obtained the data

$$13 \quad 10 \quad 20 \quad 25$$

representing their weight loss in pounds. He assumes that the weight-loss data for customers are normally distributed.

a) Formulate appropriate null and alternative hypotheses for Fred to use, and enter them in the boxes:

H_0:

H_a:

b) State the rejection rule.

Rejection rule:

c) Compute the value of the test statistic.

Value of the statistic:

d) Should Fred reject the null hypothesis at the 5% level of significance? Explain.

Decision (circle one):   Do not reject H_0 Reject H_0

e) What can you say about the P-value for part (d) (circle one, *and explain*):
   (1) The P-value is less than 0.01.
   (2) The P-value is between 0.01 and 0.025.
   (3) The P-value is between 0.025 and 0.05.
   (4) The P-value is between 0.05 and 0.1.
   (5) The P-value is greater than 0.1.

Problem 10. (20 pts) After losing a game with friends, Alice suspects that the die which was used was not fair. She suspects that the probability of "1" appearing is not $1/6$ and she decides to test this by rolling the die 300 times and using a level of significance of 5%.

a) Formulate appropriate null and alternative hypotheses for Alice to use, and enter them in the boxes:

H_0:

H_a:

b) State the rejection rule:

Rejection rule:

c) After rolling the die 300 times, she noticed that "1" appeared 38 times. Compute the value of the test statistic.

Value of the statistic:

d) Should Alice reject the null hypothesis at the 5% level of significance? Explain.

Decision (circle one):   Do not reject H_0 Reject H_0

e) Find the P-value.

P-value: