MATH 218 **FINAL EXAM** May 7, 2001

Directions

Every numerical answer should be simplified to a fraction or a decimal. You must show your work and justify your methods to obtain full credit. Use the continuity correction whenever appropriate. If you can't do one part of a problem but need that answer later, guess an answer and use that guess for the later part. The exam is worth a total of 200 points.

1. (15 points) Consider the table below of 100 stocks classified according to type and performance over the last year.

	Technology	Health	Utility	Total
Earnings $> 10\%$	10	13	7	
$0 <$ Earnings $\leq 10\%$	20	12	3	
Earnings ≤ 0	30	0	5	
Total				

   (a) (5 points) Find the probability that a randomly selected technology stock earned more than 10%.

   (b) (5 points) A stock is selected at random. Are the events {Is a technology stock} and {Earned $> 10\%$} independent?

   (c) (5 points) Find the probability that an investor who randomly selected 4 stocks at the beginning of last year had exactly one earn $> 10\%$, and three earn between 0 and 10%.

2. (15 points) The Payless Insurance Company has three types of customers; high risk, medium risk and low risk. 20% of its customers are high risk, 30% are medium risk and 50% are low risk. 14.8% of all their customers had at least one accident this last year but the lower risk customers were less prone to accidents. Indeed, 16% of their medium risk customers and only 10% of their low risk customers had at least one accident this last year.

   (a) (5 points) Draw a tree diagram that describes this situation filling in as much information as you can. *(Hint: start the tree with the events related to the three types of customers).*

   (b) (5 points) A randomly selected customer had an accident this last year. What is the probability that he/she is a medium risk customer?

   (c) (5 points) A high risk customer is selected at random. What is the probability that he/she had at least one accident this last year?

3. (10 points) The scores on a certain exam were normally distributed with a mean of 72 points and a standard deviation of 5 points.

   (a) (5 points) What proportion of students scored between 67.1 and 77.2 points?

   (b) (5 points) The top 15% of students are to receive an "A". What is the lowest score to get an "A" on that exam?

4. (25 points) The Poisson casino in Las Vegas is considering a new gambling game where a single fair die is rolled and the pay-out is +3 (i.e. gambler wins \$3) if any of the numbers 4, 5, or 6 are rolled, −5 (i.e. gambler loses \$5) if 1 or 2 is rolled and, 0 if a 3 is rolled.

   (a) (10 points) Let X denote the amount won when the game is played one time. Find the expected value and standard deviation of X. *(Hint: You may find it helpful to first fill in the blanks below in the probability distribution table of X.)*

r	-5	0	3
$P[X = r]$			

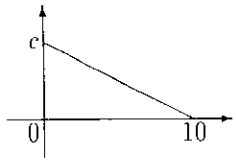
   (b) (5 points) If the gambler plays the game twice, what is the probability that the total winnings are greater than 0?

   (c) (10 points) If the gambler plays the game 100 times, find the approximate probability that the total winnings are greater than 0.

5. (15 points) The waiting time, T, in minutes, for a train at a particular station is a random variable with probability density function

$$f(t) = \begin{cases} c(1-0.1t) & \text{if } 0 \leq t \leq 10, \\ 0 & \text{otherwise.} \end{cases}$$

The graph of f is shown below.



(a) (5 points) Find the value of c.

(b) (5 points) Find the probability that a randomly selected passenger waits less than 5 minutes.

(c) (5 points) You have been waiting for the train for 5 minutes. What is the probability that you will have to wait for at most 3 more minutes?

6. (15 points) 10% of all people contacted by a life insurance company buy insurance from that company. Using an appropriate approximation find the probability that out of 600 people contacted by the company 55 or fewer buy life insurance. You may assume that the 600 people form a simple random sample.

7. (15 points) A downtown hotel fills an average of 71 rooms per weekend with a standard deviation of 15 rooms. Suppose that the distribution of the number of rooms filled is (approximately) normal.

(a) (5 points) The hotel schedules adequate staff to handle a response of 80 rooms. What is the probability that additional staff will be needed on one particular weekend?

(b) (10 points) The hotel runs a special promotion to try to fill rooms that aren't usually occupied on weekends. The promotion manager reviews the weekend demand over a 10-week period. If the average demand over this period exceeds 80, the manager will increase the scheduled staff. Suppose the promotion is not successful and the distribution of the number of rooms filled remains unchanged. What is the probability that the average demand over the 10-week period will exceed 80? You may regard the 10-week period as a random sample.

8. (25 points) A real estate agent would like to enter the Malibu luxury housing market and is interested in assessing the mean value of homes in this category. To do so, she randomly selects four homes. These homes are valued at 5,6,7, and 4 million dollars. Assume that the distribution of values is normal.

(a) (10 points) Provide point estimates for the mean and standard deviation of the value of Malibu luxury homes.

(b) (10 points) Determine a 95% confidence interval for the mean value of such homes.

(c) (5 points) Determine how many additional observations the agent needs in order to estimate the mean value of Malibu luxury homes to within $\pm.5$ million dollars with 95% confidence.

9. (15 points) The NIH is trying to assess the prevalence of Romulan Fever. They randomly select 100 members of the population and observe them for one year. Six of the 100 people observed develop Romulan Fever. Find a 95% confidence interval for p, the true proportion of the whole population that develops Romulan Fever in a one year period.

10. (20 points) Before last year's crisis in the Japanese stock market, the average foreign investment per company among *all* Japanese technology companies was 250 million Yen. The Finance minister of Japan wants to know if this has declined. However, it is intrusive and difficult to obtain information from all companies mid-year, so he has his assistant consider a random sample of 15 technology companies and perform a hypothesis test.

(a) (5 points) State the appropriate null and alternative hypotheses. Be sure to explain your reasoning.

(b) (10 points) Choose a test statistic and rejection rule to test the null hypothesis at the 5% significance level, using the random sample of 15 technology companies. You may assume that investments in Japanese technology companies are normally distributed.

(c) (5 points) After some armtwisting, the assistant determines that the *current* average foreign investment for the 15 companies selected is 238.5 million Yen, with a sample standard deviation $s = 20$ million Yen. Should the null hypothesis be rejected? *Explain your reasoning.*

11. (20 points) Charles and David are playing a game that involves rolling a die. They suspect that the die might be weighted so that the chance of rolling a 6 is not equal to $\frac{1}{6}$. They decide to each perform an hypothesis test of $H_0 : p = \frac{1}{6}$ versus $H_a : p \neq \frac{1}{6}$.

    (a) (10 points) Charles rolls the die 50 times and gets 13 sixes. Calculate his P-value.

    (b) (5 points) Should Charles reject H_0 at the 10% significance level? Explain.

    (c) (5 points) David rolls the die a much greater number of times but also gets 26% of the rolls to be sixes. Is his P-value greater than, less than, or equal to Charles' P-value? Explain.

12. (10 points)

    (a) (5 points) A certain disease occurs in 2% of the population. It is suspected that higher levels of chromium in the water may increase the rate of occurrence of this disease. One hundred different locations are identified that have higher levels of chromium in the water. At each of these locations a test is done of $H_0 : p = 0.02$ versus $H_a : p > 0.02$ where p represents the proportion of the population in that location that acquire the disease. The tests are performed at the 5% level. Suppose that, in actual fact, the higher levels of chromium in the water have no effect on the rate of occurrence of this disease and that $p = 0.02$ in each of these locations. In how many of the locations would we expect the tests to reveal incorrectly that the alternative hypothesis, $p > 0.02$, be true? Explain.

    (b) (5 points) Alex and Elizabeth each perform a test of $H_0 : \mu \leq 30$ versus $H_a : \mu > 30$. Alex uses a sample of size 50 and Elizabeth uses a sample of size 200. Remarkably, they get the same P-value. Who has more compelling evidence for H_a or is their evidence equally compelling? Explain.